Priced Out: Financing Frictions and Startup Mortality*

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Abstract

Startups raise equity in a sequence of financing rounds, and most of them are led by an investor new to the firm, who bargains with its current owners over the share of the firm it receives for the capital it provides. We ask whether investors' bargaining power causes inefficient startup failures and, if so, how many. We model equity financing as a dynamic game and show that inefficient failures arise precisely when future investors have bargaining power. Bargaining power at the current round only changes how the round's surplus is divided. However, those who decide whether to fund the firm today treat the surplus that future investors will capture as a loss, so they let firms that are worth funding fail. Using PitchBook data on US venture-backed startups, we find that in otherwise similar rounds, owners keep 1.85% less of the firm when a new investor, rather than a current owner, leads the round. The model identifies this gap as new investors' bargaining power. Although small, it causes 7.1% of startup failures, and six in ten of these firms would otherwise go public or be acquired.

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1 Introduction

Venture-backed startups play an outsized role in the US economy. Venture-backed companies represent half of the US public companies founded in the past 50 years and account for 41% of the market value and 62% of the R&D spending of all US public companies (Gornall and Strebulaev, 2021). With few assets to pledge as collateral and no stable cash flow to service debt, these firms can rarely borrow (Hall and Lerner, 2010). Instead, they raise equity from investors in a sequence of financing rounds.

The lead investor sets the price of each round, the share of the firm sold for the capital raised. In 76% of the rounds after the first, the lead investor is new to the firm and bargains over this price with the firm's current owners. At the same time, many venture-backed startups fail. Of the US firms that raised a first priced round between 2005 and 2018, 39% had failed by October 2025. In this paper, we ask whether investors' bargaining power causes inefficient startup failures and, if so, how many.

We answer this question by modeling equity financing as a dynamic game, which we estimate on a panel of US venture-backed firms from PitchBook. In the model, a firm raises capital in a sequence of rounds until it pays a single, stochastic dividend. At each round, either its current owners decide whether to fund it or a new investor does, bargaining with the owners over the share of the firm it receives. We show that inefficient failures arise precisely when future investors have bargaining power. In the data, owners keep 1.85% less of the firm when a new investor leads a round than when one of the current owners leads it, which the model identifies as new investors' bargaining power. This bargaining power is small, but it causes 7.1% of startup failures, and six in ten of these firms would otherwise go public or be acquired.

The model consists of a firm that pays a single stochastic dividend at a random date. The size of the dividend is observable by every investor and evolves as a geometric Brownian motion. Until the dividend is paid, the firm needs new capital from time to time, and these financing needs arrive at a constant rate. At each of them, it must raise a new round of equity or go bankrupt.

Rounds are of two types: inside rounds, at which the firm's current owners decide whether to fund it, and market rounds, at which a new investor decides and captures part of the surplus the round creates. At an inside round, money is all the firm needs, and the owners either put it in or let the firm fail. At a market round, the new investor decides whether to fund the round and bargains with the owners over the share of the firm it receives. We define new investors' bargaining power as the share of the round's surplus the new investor captures. A market round arises when either the current owners cannot fund the firm themselves or the firm needs an input that they lack, such as expertise in a new market or a network of

customers. A new investor is always available, so a firm never fails for lack of investors.

We show that the equilibrium of this game is unique and takes the form of a threshold: at every financing need, the round is funded if and only if the dividend is high enough. A new investor takes into account what it expects future investors to do, and that risk is reflected in the share it asks for. If it expects future investors to be less likely to fund the firm, the firm is worth less to it, and it asks for a larger share of the firm in exchange for its capital. Hence, there are no self-fulfilling failures, in which a firm fails only because investors expect it to fail.

New investors' bargaining power raises the threshold. At the current round, it only changes how the surplus is divided between the owners and the new investor, so it does not affect whether the round is funded. However, everyone who holds the firm's equity after this round anticipates that future new investors will capture part of the surplus of every market round to come. This lowers the value of their shares and therefore the surplus of the current round, so the round is funded only at a higher dividend. Because this happens at every remaining market round, a small bargaining power at each round can add up to a large loss for the firm's shareholders.

We then characterize the efficient funding policy, the one that maximizes the sum of the payoffs of the firm's owners and of every investor who funds it in the future. We show that the efficient funding policy also takes the form of a threshold, which we call the efficient threshold, and that it coincides with the equilibrium threshold precisely when new investors have no bargaining power. When they do, the equilibrium threshold lies above the efficient one.

The inefficiency arises because those who decide whether to fund the firm today treat the surplus captured by future investors as a loss. This surplus is not a loss, though, but a transfer to the investors who fund the firm later, which those deciding today fail to internalize. As a result, a firm whose dividend lies between the two thresholds when a financing need arrives is worth funding but fails. We refer to these failures as inefficient.

We then ask what fraction of all failures are inefficient. We consider a continuum of firms that enter the economy over time with dividends drawn from an entry distribution, and we study the economy in its stationary state. We show that the fraction of failures that are inefficient has a closed form, which depends on the efficient and equilibrium thresholds only through their ratio and not on the form of the entry distribution. Not all of these firms would go public or be acquired if new investors had no bargaining power, since some would later fall below the efficient threshold and fail anyway. We show that the share of these firms that would succeed also has a closed form that does not depend on the entry distribution.

We estimate the model using a panel of venture-backed firms from PitchBook. The panel covers 41,722 US firms that raised their first priced round between 2005 and 2018 and follows each of them through October 2025, by which time the firm has gone public, been acquired

or bought out, failed, or is still active. We match these firms to employment records from Revelio Labs, which provide a proxy for how the firm's prospects change between two rounds. We classify a round as a market round when a new investor leads it and as an inside round when every lead investor already owns part of the firm. Since the lead investor sets the price of the round, a market round is priced by a new investor who bargains with the owners, and an inside round by the owners themselves, as in the model.

The share of the firm that the owners keep after each type of round identifies new investors' bargaining power. In the model, the share sold at an inside round only pays for the capital raised, while at a market round the new investor also receives its part of the round's surplus. Given the size of the round and the value of the firm, the owners therefore keep less of the firm after a market round, and the gap measures new investors' bargaining power. In the data, we compare market and inside rounds of the same firm, controlling for the size of the round, the firm's previous valuation and its headcount growth, as well as for the firm's stage and market conditions.

Owners keep 1.85% less of the firm after a market round than after an inside round, which implies that new investors capture 1.85% of the surplus of a market round. This means that if an inside round sells 20% of the firm, a comparable market round sells about 21.5%. We rule out several alternative sources of this gap. New investors may know less about the firm than its owners and ask for a larger share to protect themselves. This concern should be strongest for young firms, but the estimate does not vary significantly with the age of the firm. New lead investors may also differ from returning ones, for example in their reputation or the services they bring, but comparing rounds led by the same investor, as a new investor at some firms and as a returning one at others, gives a similar estimate. Nor does the gap reflect the other terms of the round. Controlling for liquidation preferences, participation and anti-dilution protection, which change what a given share of the firm is worth, leaves the estimate unchanged.

The remaining parameters govern how often financing needs and exits arrive and how the dividend evolves. Exit values are selected, since only firms that survive every financing need reach exit, so we estimate the drift and volatility of the dividend by simulated method of moments, using the model to account for this selection. In the estimated model, new investors' bargaining power raises the threshold by 3.3%. As a result, it causes 7.1% of all failures, and six in ten of these firms would otherwise go public or be acquired.

Related Literature. This paper contributes to several strands of the literature.

First, this paper relates to the literature on financing risk in staged equity financing (see Nanda and Rhodes-Kropf, 2017; Parry, 2025; Chen, 2026). Nanda and Rhodes-Kropf (2017) define financing risk as the possibility that a viable firm fails because investors expect future

investors not to fund it, and obtain it in a model in which the firm's exit value depends on whether it can raise funds. They also show, in an appendix, that the risk disappears in a simpler model when the stake sold at each round adjusts to the firm's prospects, because a firm worth funding can always offer a larger stake. We prove the general version of that observation. With stochastic fundamentals that are exogenous to financing, a new investor at every market round, and stakes set by bargaining rather than by competition, the equilibrium is unique and coordination among investors cannot cause failure, and we show that neither the conjecture that no future round is funded nor the conjecture that every future round is funded is self-fulfilling. What survives is the price of outside capital, which their competitive investors do not charge.

Parry (2025) is the closest quantitative work. He estimates a model in which entrepreneurs and investors meet in a frictional matching market and investors stage capital to limit losses from entrepreneurs who hide failure. His financing risk is the risk of not finding an investor, and ours is the price that the investor found charges. His estimate is a share of startups and ours a share of failures, so the two are not directly comparable, but they are complements: a thin market for outside capital lowers his matching rate and could also change the bargaining power of the investors that startups find.

Chen (2026) constructs a text-based, firm-level measure of financing risk, the expectation that a startup will not be able to raise its next round, and shows that it reduces innovation and growth and raises failure even among recently funded firms. In our model that probability is an equilibrium object, the chance that the next financing need arrives while the dividend is below the funding threshold.

Second, this paper relates to the literature on staged financing and hold-up. Staged financing itself is the response to two problems, hold-up by a founder who cannot commit her human capital to the venture (see Hart and Moore, 1994; Neher, 1999) and learning about a project whose success and completion time are uncertain (Bergemann and Hege, 2005). We take staging as given and study the market it creates, in which the hold-up runs the other way, of the firm by the investor who supplies the next round rather than of the investor by the founder. Hellmann and Thiele (2015) model this hold-up in a market with two stages, in which the later investor no longer needs the earlier one, and derive its effect on valuations and exit rates. Our market rounds are a stationary, repeated version of it, and we measure its cost. Admati and Pfleiderer (1994) study how an inside investor prices the rounds that follow its entry and show that a fixed-fraction contract removes its incentive to misprice them, which is the reference point for our treatment of inside rounds as priced at the equity value.

Third, this paper relates to the literature on investors' bargaining power in the market for venture capital. Michelacci and Suarez (2004), Inderst and Müller (2004), Silveira and Wright (2016) and Ewens, Gorbenko and Korteweg (2022) model this market as one of search

and matching between entrepreneurs and investors, in which the surplus of a match is split by bargaining, so the share an investor obtains reflects how long the entrepreneur would take to find another one. In Inderst and Müller (2004), investors' bargaining power is itself determined by the relative scarcity of venture capital, and Jovanovic and Szentes (2013) obtain rents for investors without search frictions, from the scarcity of venture capitalists in a competitive equilibrium. Ewens, Gorbenko and Korteweg (2022) estimate, on first rounds, that investors use their bargaining power to obtain terms that lower the value of the startup. We do not model where bargaining power comes from. We take it as the share of the surplus that a new investor captures at every market round, identify it from the difference between the shares sold at market and inside rounds, and measure its cost in startup failures. Our estimate comes from later rounds, relative to the rounds the owners price themselves, and shows that bargaining power affects not only how the surplus of a round is split but, through future rounds, which startups survive.

Finally, this paper relates to the literature on rollover risk and dynamic runs (see Acharya, Gale and Yorulmazer, 2011; He and Xiong, 2012*a*,*b*; Brunnermeier and Oehmke, 2013; Schroth, Suarez and Taylor, 2014; He and Milbradt, 2016). Our model is built on He and Xiong (2012*a*), in which a firm with a time-varying fundamental repeatedly rolls over debt at fixed terms, so a maturing creditor bears the risk that the creditors maturing after it do not roll over. They show that when a shock makes rollover harder, each creditor raises its rollover threshold not only in response to the shock but also in response to the other creditors raising theirs, and this rat race amplifies its effect. We repeat their exercise in our model and show that there is no rat race in staged equity financing, since the responses of investors to one another dampen the effect of such a shock rather than amplify it. That equity claims are not exposed to runs is an old observation (Jacklin, 1987). We show that staged equity financing is not exposed to runs either, but that it still leads to inefficient failures, which come from the bargaining power of future investors.

Outline. The rest of the paper is organized as follows. Section 2 develops the model of staged equity financing, defines the equilibrium and shows that it is unique and takes the form of a funding threshold. Section 3 defines the efficient benchmark, shows that the equilibrium attains it when new investors have no bargaining power, and derives in closed form the share of failures that new investors' bargaining power causes and the share that removing it would prevent. Section 4 describes the data and maps them to the model. Section 5 estimates the parameters, and Section 6 reports the cost of bargaining power at the estimates. Section 7 concludes. Proofs are in Appendix A.

2 Model

This section develops a model of staged equity financing. A single firm with a stochastic dividend must repeatedly raise capital to stay alive, and each round is funded either by the firm's current owners or by a new investor from the market. A new investor bargains with the current owners over the share of the firm it receives in exchange for the capital it provides. We will refer to θ, the share of the surplus created by the round that the new investor captures, as new investors' bargaining power. The next subsection describes the firm and its investors. The financing game that follows specifies who moves at each round, what they can do, and on what terms. We then define the equity value, the value of all of the firm's equity once a round is funded, which both the owners and a new investor weigh against the cost of the round, restrict attention to symmetric Markov equilibria, and show that the equilibrium is unique and takes the form of a threshold. The firm raises another round if and only if its dividend exceeds a cutoff, and new investors' bargaining power raises that cutoff.

2.1. Environment

The economy is set in continuous time over an infinite horizon. We study a single firm in partial equilibrium, with investors who are risk neutral and discount the future at rate ρ.

The firm generates no cash flow until a random date, at which it pays a single, one-time dividend to its equity holders and is dissolved. The size of the dividend y_t is publicly observable[1] and follows a geometric Brownian motion

$$\frac{\mathrm{d}y_t}{y_t} = \mu\,\mathrm{d}t + \sigma\,\mathrm{d}Z_t$$

where Z_t is a standard Brownian motion, μ is the drift and σ the volatility. A higher y_t means a larger payout to whoever holds equity when it is paid. Modeling the dividend as a process, rather than as a fixed number, captures the nature of a startup. What the firm will be worth when its value is realized depends on the outcome of a development process that is still under way, so the firm an investor sees today can look very different one year from now.

Two Poisson processes govern the events in the firm's life. The first is a payout shock, which arrives at rate ϕ. When it arrives, the firm pays the dividend y_t to its equity holders, pro rata,[2] and is dissolved. The second is a financing need, which arrives at rate λ and forces

[1] The assumption that the dividend is publicly observable is strong. It spares the model the complications that would arise if investors held private information about the dividend. However, the model would not change if the dividend is unobservable and every player observes the same noisy public signal of it, since every decision then conditions on the signal in place of the dividend. He and Xiong (2012*a*) make the same assumption for the same reason.

[2] Pro rata means in proportion to the share of the firm's equity each holder owns, so a holder of a share s of

the firm to raise

$$x(y_t) = \alpha y_t + x_0$$

in new equity or go bankrupt, in which case every equity claim pays zero. Both processes are independent of each other and of Z_t.

The financing need captures the fact that a startup burns cash between rounds and must return to the market at dates it does not fully control. The development process is uncertain, and so is the amount of money it takes to keep the firm alive until the payout. A setback, a longer trial, or a product that needs one more iteration before it sells all mean that the cash raised at the last round runs out earlier than planned, and the firm must raise again or fail. The date at which the money arrives is uncertain on the investors' side as well. A round takes time to close, through due diligence, the negotiation of the terms and the search for investors to fill it, and how long it takes is not under the firm's control either. Modeling financing needs as arrivals of a Poisson process captures both sources of uncertainty in the simplest way, with a single parameter for how often the firm has to return to the market. The size of the round has a fixed component $x_0 > 0$ and a component αy_t, with $\alpha \geq 0$, so we allow for the possibility that larger firms raise larger rounds.

2.2. The Financing Game

Each financing need is a round, which can be one of two types. With probability $1 - p$ the round is an inside round, in which the current owners, the investors who hold the firm's equity, decide whether to put in $x(y)$ themselves or let the firm fail. With probability p the round is a market round, in which the current owners cannot fund it and the firm must raise $x(y)$ from a new investor. The type of a round is drawn independently of everything else, and p measures how often the firm has to go back to the market.

The two types of round capture the idea that a round is not only a transfer of money. As the development process unfolds, the firm may find that it needs something its current owners cannot provide, expertise in a market it is entering, a network of customers, or operating experience it lacks, and the way to bring it in is to sell equity to an investor who has it.

The current owners may also be unable to fund the round for reasons of their own. Venture funds specialize by stage, and a fund that invests only in seed and early-stage companies does not lead the later rounds of a firm it backed at the start (Gompers et al., 2020). A fund also has a finite life, typically ten years (Gompers et al., 2020), and its agreement with its limited partners usually caps the share of the fund's committed capital that can be invested in any one company (Gompers and Lerner, 1996), so a firm whose investors have reached the end of their investment period, or their cap, must find a new one.

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the firm receives $s y_t$.

An inside round is one in which money is all the firm needs, and its owners can supply it. A market round is one in which the firm must go to the market, and it raises the capital and, when it needs one, the missing input in the same transaction. Which of the two a given round turns out to be is a feature of the development process or of the investors' mandates, not a choice of the firm, which is why we model it as an independent draw.

The players are the founder and a sequence of outside investors, indexed by $i = 0, 1, 2, \ldots$, with investor 0 the founder, who owns the firm at time 0. Let T_i be the arrival time of the i-th financing need. If round i is a market round, investor i is matched to the firm and decides whether to fund it. If round i is an inside round, investor i never enters. A new investor therefore decides at most once. If it funds, it receives equity and joins the current owners, who act as a coalition at every inside round. We treat this coalition as a single player, the owners.

At round T_i one player decides whether the round is funded, the owners at an inside round and investor i at a market round. We will refer to this player as the active player. The active player observes the dividend y_{T_i} and chooses one of two actions, fund or walk. Walk means the same at every round. The round fails, the firm goes bankrupt, and every equity claim pays zero. Fund puts $x(y_{T_i})$ into the firm, and its terms depend on the type of the round.

If the owners fund an inside round, they put in $x(y_{T_i})$ pro rata, each in proportion to its share of the firm's equity, so every owner keeps the share it held before the round.[3] If investor i funds a market round, it pays $x(y_{T_i})$ for a stake s_i of the firm's equity, and the share of every owner is diluted by the factor $1 - s_i$. The stake s_i is set by Nash bargaining between investor i and the owners. The outcome of the bargain depends on what the firm is worth once the round is funded, which in turn depends on how future rounds will be played, so we solve the bargaining problem in the next subsection, after defining the equity value.

The game ends at whichever of two events comes first. At the payout time τ_ϕ, the equity holders split y_{τ_ϕ} pro rata. At the first unfunded round, the firm goes bankrupt and every equity claim pays zero.

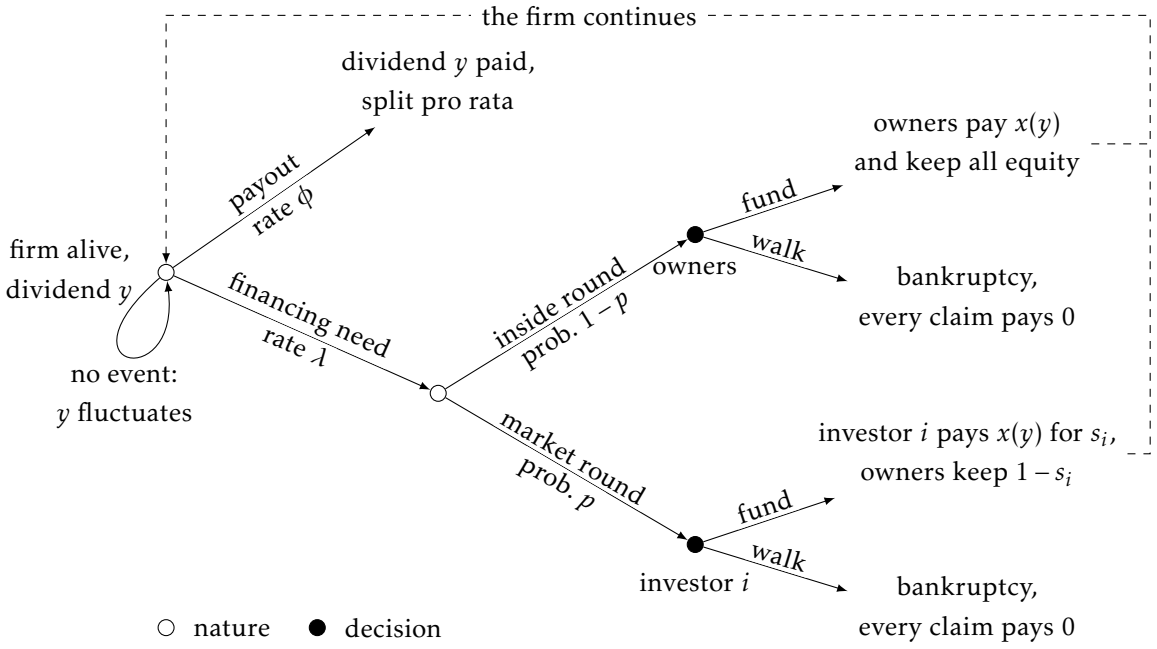
We restrict attention to Markov strategies, in which the action of the active player depends only on the dividend at the round. A strategy for investor i is then a measurable funding set $F_i \subseteq (0, \infty)$, and a strategy for the owners is a measurable funding set $F^o \subseteq (0, \infty)$. Investor i funds its market round if and only if $y_{T_i} \in F_i$, and the owners fund an inside round at T_j if and only if $y_{T_j} \in F^o$. Every new investor faces the same problem, so we look for symmetric profiles, in which every investor uses the same funding set F.[4] We will refer to a pair (F, F^o) as a symmetric Markov profile.

[3] Appendix A.1 models an inside round as a contribution game among the owners. It shows that how they divide $x(y_{T_i})$ does not affect the payoff of any of them, and that pro rata contributions are an equilibrium of that game if and only if a single player holding all of the equity would fund the round.

[4] Symmetry applies only to the new investors. We allow the owners to use a funding set F^o different from F. That the two sets coincide in equilibrium is a result, which we establish in Section 2.4, not an assumption.

Figure 1 summarizes the game.

Figure 1: The financing game



2.3. Valuation and the Terms of a Market Round

Fix a symmetric Markov profile (F, F^o). Under it, whether each future round is funded depends only on the dividend at that round, so the firm's equity has a well defined value that depends only on the current dividend, which we will refer to as the equity value. It is what the owners weigh against the cost of an inside round, and what investor i and the owners bargain over at a market round, so we define it first and then return to the bargaining problem.

Consider the investors holding the firm's equity at time t, when the dividend is $y_t = y$. Between t and the payout they are diluted at every market round that is funded, they pay the cost of every inside round that is funded, and they receive nothing if any round is left unfunded. Therefore, the equity value at t is

$$W^{F,F^o}(y) = \mathbb{E}\Big[e^{-\rho(\tau_\phi - t)}\, y_{\tau_\phi}\, D\big(t,\tau_\phi\big) S\big(t,\tau_\phi\big) - \sum_{\substack{\text{inside } j \\ t<T_j\le\tau_\phi}} e^{-\rho(T_j - t)}\, x\big(y_{T_j}\big) D\big(t,T_j\big) S\big(t,T_j\big) \;\Big|\; y_t = y \Big] \quad (1)$$

where D is the dilution factor and S the survival factor

$$D(t,u) = \prod_{\substack{\text{market } j \\ t<T_j\le u}} \big(1 - s_j\big), \qquad S(t,u) = \prod_{\substack{\text{inside } j \\ t<T_j\le u}} \mathbb{1}_{\left\{y_{T_j}\in F^o\right\}} \cdot \prod_{\substack{\text{market } j \\ t<T_j\le u}} \mathbb{1}_{\left\{y_{T_j}\in F\right\}}$$

The first term in Eq. (1) is the discounted dividend. The dilution factor accounts for the fact that the holders at t give up a stake s_j of the firm at every market round funded between t and u, and pay nothing for it. For example, suppose the next round is a market round at which the firm sells 30% of its equity to the new investor. The holders at t keep 70% of the firm, and if the round after it is also a market round at which the firm sells 30%, they keep 70% of that 70%, which is 49%.

The survival factor equals one if every round between t and u is funded under (F, F^o) and zero otherwise. A single unfunded round sets it to zero, so every round exposes the firm to financing risk. The second term is the cost of the inside rounds between t and the payout. The owners bear it in full at the time, and the equity holders at t bear the fraction $D\big(t, T_j\big)$ of it that matches the equity they still hold when that round happens. The expectation is over the stochastic dividend, the Poisson processes governing the payout and financing needs and the round types, conditional on the current dividend $y_t = y$.

We now return to the terms of a market round. Consider a market round at which the dividend is y. Once the round is funded, the firm continues from the same dividend, so $W^{F,F^o}(y)$ is the equity value once investor i's capital is in. If the round is funded, the owners keep a share $1 - s_i$ of that value and investor i receives a stake s_i in exchange for $x(y)$. If the two sides do not agree, the round is unfunded, the firm goes bankrupt, and both sides receive zero. The stake $s_i = s(y)$ solves the Nash bargaining problem[5]

$$s(y) = \underset{s\in[0,1]}{\arg\max} \Big[\underbrace{(1-s)\, W^{F,F^o}(y)}_{\text{current owners}}\Big]^{1-\theta} \Big[\underbrace{s\, W^{F,F^o}(y) - x(y)}_{\text{new investor}}\Big]^{\theta}$$

where $\theta \in [0,1]$ is the bargaining weight of the new investor, which we refer to as new investors' bargaining power. If the round is funded, it creates a total surplus of $W^{F,F^o}(y) - x(y)$, and the solution to the problem above gives investor i a fraction θ of the total surplus:

$$s(y)\, W^{F,F^o}(y) - x(y) = \theta\Big(W^{F,F^o}(y) - x(y)\Big) \tag{2}$$

We can solve Eq. (2) for the stake itself:

$$s(y) = \theta + (1-\theta)\,\frac{x(y)}{W^{F,F^o}(y)} \tag{3}$$

which is a weighted average of one and of the fraction of the firm that the new capital

[5]The Nash bargaining solution is the limit of a noncooperative game in which the two parties alternate offers indefinitely under the risk that negotiations break down (Binmore, Rubinstein and Wolinsky, 1986). In that limit the disagreement point is the payoff at breakdown, here bankruptcy, and the asymmetric weight θ reflects an asymmetry in the bargaining procedure, such as the time each side waits after making an offer or the probability of being the one to make it.

represents. The two limiting cases show what bargaining power does. When $\theta = 1$, the new investor takes the whole firm in exchange for the capital, regardless of the equity value or of the size of the round, and the owners are left indifferent between funding and bankruptcy. When $\theta = 0$, the new investor receives exactly the stake $x(y)/W^{F,F^o}(y)$ that its capital represents, so it pays what its claim is worth and captures no surplus. Every $\theta > 0$ moves the stake toward one, so the new investor pays less than its claim is worth and captures a share θ of the surplus.

At a market round with dividend y, the expected payoff to investor i if it funds is

$$\mathbb{E}\Big[U_{i,T_i} \,\Big|\, y_{T_i} = y, \text{fund}\Big] = s(y)\, W^{F,F^o}(y) - x(y)$$

Since walking pays zero, investor i's best response is to fund if and only if the stake it receives is worth at least the capital it puts in

$$y \in F_i \quad \Longleftrightarrow \quad s(y)\, W^{F,F^o}(y) \geq x(y) \tag{4}$$

where we adopt the convention that a player funds when indifferent.[6]

At an inside round with dividend y, the expected payoff to the owners if they fund is

$$\mathbb{E}\Big[U_{o,T_j} \,\Big|\, y_{T_j} = y, \text{fund}\Big] = -x(y) + W^{F,F^o}(y)$$

since they hold all of the firm's equity before and after the round. Walking again pays zero, so the owners' best response is

$$y \in F^o \quad \Longleftrightarrow \quad W^{F,F^o}(y) \geq x(y) \tag{5}$$

The next subsection defines the equilibrium as the funding sets consistent with these best responses, shows that the two sets coincide, and shows that the equilibrium is unique and takes the form of a funding threshold.

2.4. Equilibrium

We can now define the equilibrium.

Definition 1 (Symmetric Markov equilibrium)**.** A symmetric Markov profile (F, F^o) is a symmetric Markov equilibrium if

$$F = \Big\{y : s(y)\, W^{F,F^o}(y) \geq x(y)\Big\}, \qquad F^o = \Big\{y : W^{F,F^o}(y) \geq x(y)\Big\}$$

[6]The convention matters only when the stake is worth exactly the capital, which, when $\theta = 0$, happens at every dividend at which the round is funded, since a new investor with no bargaining power breaks even whenever it funds.

The definition states that F and F^o are a fixed point. A profile (F, F^o) determines the equity value W^{F,F^o} through Eq. (1). Given that equity value, Eqs. (4) and (5) define a funding set for a new investor and one for the owners, and the profile is an equilibrium when the funding sets it induces in this way are F and F^o themselves.

The two conditions in Definition 1 are not necessarily the same. A new investor compares the value of its stake with the capital it puts in, whereas the owners compare the value of all of the equity with it. The next result shows that the bargaining solution makes the two comparisons equivalent.

Lemma 1 (Who funds does not matter)**.** *In any symmetric Markov equilibrium, the funding sets of a new investor and of the owners are equal,*

$$F = F^o = \left\{y : W^{F,F^o}(y) \geq x(y)\right\}$$

so a round is funded at the same dividends whoever is asked to fund it.

Proof. From the Nash bargaining solution in Eq. (2), we have

$$s(y)\, W^{F,F^o}(y) - x(y) \geq 0 \quad \Longleftrightarrow \quad \theta\left(W^{F,F^o}(y) - x(y)\right) \geq 0 \quad \Longleftrightarrow \quad W^{F,F^o}(y) \geq x(y)$$

The first condition is F in Definition 1 and the last is F^o, so the two sets are equal.[7] □

We will refer to the common set F as the funding set and write W for the equilibrium equity value $W^{F,F}$.

Lemma 1 has an immediate consequence for the role of bargaining power. The bargaining power of the new investor at the current round does not affect whether the firm is funded today. Whoever is asked to fund the round funds it if and only if it creates a nonnegative surplus, $W(y) - x(y) \geq 0$, and θ only determines how that surplus is split between the current owners and the new investor.

Appendix A.2 shows that the equilibrium equity value satisfies the following Hamilton-Jacobi-Bellman (HJB) equation:

$$\rho W(y) = \mu y\, W'(y) + \frac{\sigma^2}{2} y^2\, W''(y) + \phi\,(y - W(y)) + \lambda\,(1 - p\theta)\,(W(y) - x(y))^+ - \lambda W(y) \qquad (6)$$

where $(u)^+ = \max\{u, 0\}$. The left side is the required return on the equity, and the first three terms on the right are the expected capital gain from the drift and the volatility of the dividend and the payout, at rate ϕ, at which the equity holders receive y and give up the equity value.

[7] The second equivalence requires $\theta > 0$. The conclusion holds for $\theta = 0$ as well. When $W^{F,F^o}(y) \geq x(y)$ the new investor breaks even and funds by our convention, and when $W^{F,F^o}(y) < x(y)$ no stake in $[0, 1]$ is worth the capital, so it walks.

The last two terms describe a financing need, which arrives at rate λ. The equity holders lose the equity value, captured by the last term, and if the round creates a positive surplus they recover a fraction of it. They recover all of the surplus at an inside round, which occurs with probability $1-p$, and a fraction $1-\theta$ of it at a market round, which occurs with probability p, so the expected fraction recovered is $1-p\theta$. If the surplus is negative the round is not funded and nothing is recovered.

The term $1-p\theta$ in Eq. (6) is how the bargaining power of future investors affects the equity value today. For the equity holders, $p\theta$ works as a proportional tax on the surplus that each future round creates, collected by the new investors who fund the market rounds. The tax lowers the equity value at every dividend. The bargaining power of the new investor at the current round only splits the surplus of that round, but the bargaining power of future investors lowers the surplus itself. Unlike a tax, what the new investors collect is not lost, but the equity holders who decide to fund today do not internalize it. This is the source of inefficiency in the model. Section 3 shows that without bargaining power the market funds exactly the firms that maximize total surplus, so the firms that fail because of the bargaining power of future investors are firms worth funding.

Two features of Eq. (6) are worth noting. First, the equilibrium is characterized by W alone. Once W is known, the funding set is $F = \{y : W(y) \geq x(y)\}$, and the stake at every market round follows from Eq. (3). Second, the frequency of market rounds and new investors' bargaining power affect the equity value only through the product $p\theta$, the expected share of the surplus that new investors receive at a financing need. A firm that goes to the market almost every round and cedes a small share of the surplus each time is in the same position as a firm that rarely goes to the market and cedes a large share when it does.

To guarantee that the equilibrium is unique whenever it exists, we impose one condition on the parameters.

Assumption 1. $\mu < \rho + \phi + \lambda p\theta$.

Assumption 1 requires that the dividend grow more slowly than the rate at which the equity holders discount it. It ensures that the operator whose fixed point is the equilibrium equity value is a contraction.

Proposition 1 (Uniqueness)**.** *Under Assumption 1, there is at most one symmetric Markov equilibrium.*

Proof: See Appendix A.3.

Proposition 1 pins down the funding set but says nothing about its form. Under one more condition on the parameters, the funding set is an interval, so the firm is funded if and only if its dividend exceeds a threshold.

Assumption 2. $\phi > \alpha(\rho + \phi + \lambda - \mu)$.

Assumption 2 bounds the slope α of how much a firm needs to raise in each round. The ratio $\phi/(\rho + \phi + \lambda - \mu)$ is the equity value, per unit of dividend, when the equity holders expect no round to ever be funded, so that the first financing need kills the firm and the only source of value is a payout that arrives before it.[8] The condition says that an additional unit of dividend raises the equity value by more than it raises the cost of the next round, even under this most pessimistic continuation conjecture. It ensures that $W(y) - \alpha y$ is strictly increasing, and it holds trivially when $\alpha = 0$.

Proposition 2 (Funding threshold)**.** *Under Assumptions 1 and 2, in any symmetric Markov equilibrium $W(y) - \alpha y$ is strictly increasing in y, so the funding set is an interval*

$$F = [y^*(\theta), \infty), \qquad W(y^*(\theta)) = x(y^*(\theta))$$

The funding threshold $y^*(\theta)$ is increasing in θ if $p > 0$, increasing in p if $\theta > 0$, increasing in α and in λ, and decreasing in σ if $p\theta < 1$.

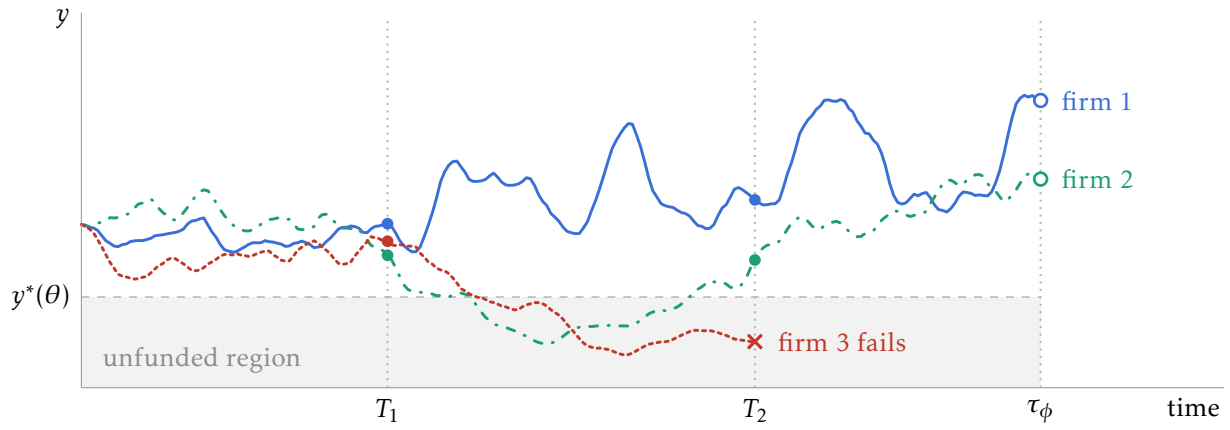
Proof: See Appendix A.4.

The threshold form of the equilibrium parallels the rollover threshold in He and Xiong (2012*a*), where each maturing creditor rolls over its debt if and only if the firm's fundamental exceeds a cutoff. Here each round is funded if and only if the dividend exceeds $y^*(\theta)$.

The threshold is not a point of no return. A firm whose dividend falls below $y^*(\theta)$ does not fail at that moment. It fails only if a financing need arrives while the dividend is still below the threshold, so the dividend can cross the threshold and recover before the next round. Figure 2 illustrates this with three paths of the dividend that share the same dates of financing needs, T_1 and T_2, and the same payout date τ_ϕ. Firm 1 stays above the threshold and is funded at both rounds. Firm 2 falls below the threshold after the first round, but no financing need arrives while it is there, and it has recovered by the time of the second round, which is funded, so it reaches the payout event like firm 1. Firm 3, the unlucky one, falls below the threshold after the first round and is still below it when the second financing need arrives. That round is not funded and the firm fails. What separates firms 2 and 3 is where the dividend stands when the financing need arrives.

[8] Appendix A.5 derives the equity value under this conjecture.

Figure 2: Sample paths of the dividend and the funding threshold



The comparative statics follow from how each parameter moves the equity value, since the threshold is the dividend at which the equity value crosses the cost of the round. A financing need is always bad news for the equity holders. At an inside round either they pay the cost of the round themselves or the firm fails. At a market round either they are diluted or the firm fails. A higher λ makes financing needs more frequent, which lowers the equity value at every dividend and raises the threshold. A higher α works in the same direction. It raises the cost of every future round, which lowers the equity value at every dividend, and it raises the cost of the current round that the equity value must cover.

The effect of σ works through option value. Volatility matters because at every financing need the equity holders can walk away instead of paying $x(y)$. Their loss is capped at the value of the equity they hold, while their gain when the dividend turns out high is not, so the payoff at a financing need is convex in the surplus $W - x$.

As previously discussed, new investors' bargaining power moves the threshold only through the equity value. If $p > 0$, some future rounds will be market rounds, and at each of them the equity holders keep a fraction $1 - \theta$ of the surplus. A higher θ lowers what they keep at every future market round, which lowers the equity value at every dividend and raises the threshold. If $p = 0$ there are no market rounds and θ plays no role.

The effect of p is similar. If $\theta > 0$, a higher p increases the chance that a future round will be a market round, and at market rounds the equity holders keep a smaller fraction of the surplus than at inside rounds. This lowers the equity value and raises the threshold. If $\theta = 0$ a market round leaves the equity holders with the same surplus as an inside round, so p plays no role.

2.5. The Absence of Runs and Self-Fulfilling Failures

This subsection shows that the uniqueness of the equilibrium is more than a technical property. It means that investors' beliefs about future rounds cannot make a firm fail. We then show that when a change in the parameters makes funding harder, such as an increase in how often financing needs arrive, the responses of investors to one another dampen its effect rather than amplify it, so there is no rat race as in He and Xiong (2012*a*).

Beliefs cannot make a firm fail because the stake sold at every round adjusts to the equity value. A belief that future rounds will not be funded lowers the equity value, but the new investor is compensated through a larger stake rather than exposed to it at fixed terms, so whether a round is funded depends only on whether the equity value covers its cost. Neither pessimism nor optimism about future rounds is therefore self-fulfilling. If the active player expects no future round to be funded, it still funds the firm whenever the dividend is high enough, and if it expects every future round to be funded, it still lets the firm fail whenever the dividend is low enough. The best response to either conjecture is a threshold, so neither conjecture is an equilibrium. Appendix A.5 shows this formally.

The absence of a rat race can be seen by repeating the experiment in Section 2.3 of He and Xiong (2012*a*). Suppose a parameter changes, say the arrival rate of financing needs rises, and consider the active player right after the change, who takes as given that every future round is funded at the old threshold. Its best response is a new threshold, which we call the individual response to the change. The equilibrium adjustment is the distance between the old and the new equilibrium thresholds.

Figure 3 compares the two in the two models. Figure 3(a) is the debt model. The figure shows the best response curves at the initial parameters, the dotted blue line, and at the new parameters, the solid red line, as a function of the conjectured threshold of the other creditors. A conjecture is an equilibrium when the best response to it is the conjecture itself, so the equilibrium thresholds are where the curves cross the 45° line. The initial equilibrium threshold is $y_{*,0}$, where the blue line crosses it, and the new equilibrium threshold is $y_{*,\infty}$, where the red line does.[9] After the change, a creditor who expects the others to still use $y_{*,0}$ as their strategy reads its best response off the red line and raises its own threshold to $y_{*,1}$, the vertical arrow. Once every creditor has done the same, each expects the others to use $y_{*,1}$, moves along the 45° line to that conjecture, and raises its threshold again, to $y_{*,2}$. This second step is a response to the other creditors' response rather than to the change in the parameter. This is the rat race, in which creditors run because other creditors run. The steps continue up the red line and converge to $y_{*,\infty}$. The individual response, from $y_{*,0}$ to $y_{*,1}$, is a fraction of the equilibrium adjustment, from $y_{*,0}$ to $y_{*,\infty}$, and the mutual responses supply the rest, which is

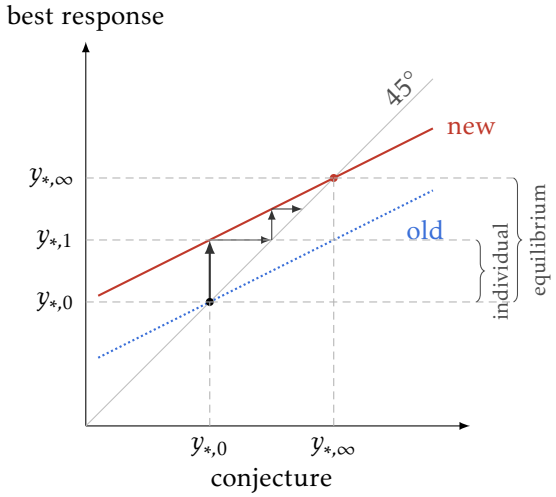
[9] The notation in panel (a) follows He and Xiong (2012*a*), who write $y_{*,0}$ for the threshold before the change, $y_{*,k}$ for the threshold after k rounds of responses, and $y_{*,\infty}$ for the new equilibrium threshold.

the rat race effect.

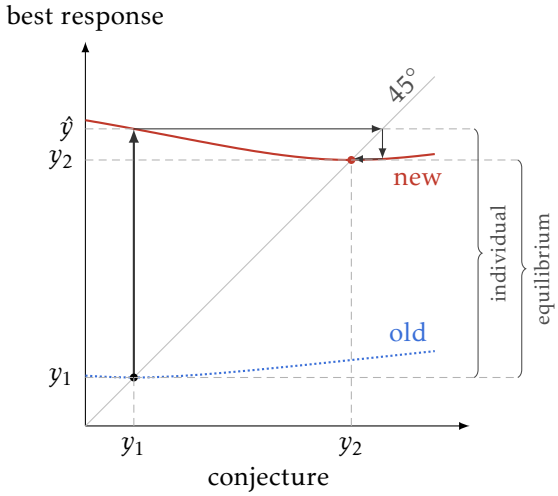
Figure 3(b) is our model, drawn at the parameter values of Table 3 for a rise in λ from 0.315 to 0.8, with the same axes and the same colors, the dotted blue line for the best response at the initial parameters and the solid red line for the best response after the rise in λ. The initial equilibrium threshold is y_1, where the blue line crosses the 45° line, and the new one is y_2. After the change, a player who expects the others to still follow y_1 as their strategy raises its threshold to $\hat{y}$, the vertical arrow, and $\hat{y}$ lies above y_2. Once every player uses $\hat{y}$, the best response to that conjecture is lower, and the steps come down the red line to y_2. The individual response, from y_1 to $\hat{y}$, exceeds the equilibrium adjustment, from y_1 to y_2. The mutual responses act in the opposite direction, dampening the first adjustment rather than amplifying it, which is why there is no rat race. Appendix A.6 proves that the individual response is at least as large as the equilibrium adjustment, for any change that raises the threshold.

Figure 3: Best responses to a change in a parameter

(a) Debt rollover, He and Xiong (2012*a*)



(b) Staged equity, this paper



3 The Cost of Bargaining Power

This section defines the cost of new investors' bargaining power. We first define the efficient benchmark for this economy as the funding policy that maximizes total surplus, the sum of the payoffs of the founder and of every investor. We then show that the market without bargaining power attains this benchmark, that is, the market with $\theta = 0$ funds exactly the firms the efficient policy funds, and its threshold $y^*(0)$ is the efficient one. With the benchmark in hand, the cost of bargaining power is the distance between the two thresholds. A firm whose dividend lies between $y^*(0)$ and $y^*(\theta)$ is worth funding and fails because of new investors'

bargaining power when a financing need arrives. The next subsection counts these firms. It introduces a population of firms and derives in closed form the share of failures that bargaining power causes and the share of failures that removing it would prevent, the two objects that the estimation later quantifies.

3.1. Efficient Benchmark

The efficient benchmark for this economy is the outcome that maximizes total surplus. The structure of the economy is that of Section 2. Financing needs arrive at rate λ, each is a market round with probability p and an inside round with probability $1-p$, a market round can only be funded by a new investor, and a round costs $x(y)$ whoever funds it. A policy specifies two things. At each financing need, it decides whether the firm is funded or fails, and at each market round, it decides the stake s_i that the new investor receives. Total surplus is the sum of the expected discounted payoffs of the founder and of every new investor, with equal weights, conditional on the current dividend $y_t = y$.

We keep the notation of Section 2.3. Let $d_j \in \{0,1\}$ be the funding decision at round j, equal to one if the round is funded and to zero if the firm fails. The survival factor is

$$S(t,u) = \prod_{\substack{\text{all rounds } j \\ t<T_j\leq u}} d_j$$

Appendix A.7 shows that total surplus does not depend on the stakes. A stake is a transfer between players who have the same linear preferences and the same discount rate, so it redistributes the payoff without changing it, and what remains once the payoffs are added is the dividend, if the firm survives to the payout, net of the cost of every funded round, whoever puts the capital in. Taking expectations, the maximized total surplus solves the following problem:

$$V(y) = \max_{(d_j)_{j\geq 1}} \mathbb{E}\Big[e^{-\rho(\tau_\phi - t)}\, y_{\tau_\phi}\, S\big(t,\tau_\phi\big) - \sum_{\substack{\text{all rounds } j \\ t<T_j\leq \tau_\phi}} e^{-\rho(T_j - t)}\, x\big(y_{T_j}\big)\, S\big(t,T_j\big) \;\Big|\; y_t = y\Big] \tag{7}$$

where the maximum is over the funding decision d_j at every financing need after t. Each decision may depend on everything observed up to its date, but since the dividend is a Markov process and the two clocks are memoryless, the efficient decision at a financing need depends only on the current dividend, which is why V is a function of y. At a financing need the efficient policy compares the value of continuing net of the cost of the round, $V(y) - x(y)$, with the value of failure, which is zero, so it funds the firm if and only if $V(y) \geq x(y)$, and V satisfies

the following HJB equation:

$$\rho V(y) = \mu y\, V'(y) + \frac{\sigma^2}{2} y^2\, V''(y) + \phi\,(y - V(y)) + \lambda\,(V(y) - x(y))^+ - \lambda V(y) \tag{8}$$

Compare this with the equilibrium HJB of the decentralized market, Eq. (6), which we restate here

$$\rho W(y) = \mu y\, W'(y) + \frac{\sigma^2}{2} y^2\, W''(y) + \phi\,(y - W(y)) + \lambda\,(1 - p\theta)\,(W(y) - x(y))^+ - \lambda W(y)$$

The two equations differ in one place. The HJB of the efficient benchmark is the market's with $p\theta = 0$. At $\theta = 0$, Assumption 1 becomes $\mu < \rho + \phi$, which implies it for every θ, and Assumption 2 does not involve θ. Under these two conditions the arguments of Propositions 1 and 2 apply to Eq. (8) without change, so V is its unique solution, $V - \alpha y$ is strictly increasing, and the efficient policy funds the firm if and only if the dividend exceeds a threshold, which we call y^{eff}. Writing $W(\cdot\,;0)$ for the equity value of the market at $\theta = 0$, we have the following result.

Proposition 3 (Efficiency)**.** *Under Assumption 2 and $\mu < \rho + \phi$, the maximized total surplus coincides with the equity value of the market without bargaining power, and the efficient policy funds exactly the firms that the market without bargaining power funds,*

$$V = W(\cdot\,;0), \qquad y^{\text{eff}} = y^*(0) \tag{9}$$

Proof. It follows from the discussion above. □

Proposition 3 makes precise the mechanism anticipated in Section 2.4. Comparing Eq. (8) with Eq. (6), at every future financing need the efficient policy counts the whole surplus of the round, whereas the equity holders keep only a fraction $1 - p\theta$ of it, since new investors capture a share θ of the surplus at every market round. What the new investors capture is a transfer and remains part of total surplus, but the equity holders who decide whether to fund the firm today do not internalize it. Their equity value is therefore below the maximized total surplus, and they let the firm fail at dividends at which the efficient policy would fund it. With the benchmark in hand, the cost of bargaining power is the distance between $y^*(\theta)$ and $y^*(0)$, which the next subsection measures.

3.2. Inefficient Failures

By Proposition 2, the funding threshold is increasing in new investors' bargaining power whenever $p > 0$, so

$$\Delta(\theta) = y^*(\theta) - y^*(0) \geq 0 \tag{10}$$

with a strict inequality if $p\theta > 0$. A firm whose dividend lies in $[y^*(0), y^*(\theta))$ when a financing need arrives is worth funding, since the efficient benchmark funds it, but is not funded in the market. It fails because of new investors' bargaining power, not because of its fundamentals. We will refer to these failures as inefficient.

We now turn to the main question of the paper: what fraction of startup failures does new investors' bargaining power cause? We consider a continuum of firms, each following the model of Section 2 with the same parameters and independent shocks, and study the economy in its stationary state. It is convenient to work with the log dividend $u = \ln y$, which by Itô's lemma follows a Brownian motion with drift

$$\mathrm{d}u_t = m\,\mathrm{d}t + \sigma\,\mathrm{d}Z_t, \qquad m = \mu - \frac{\sigma^2}{2}$$

and to write $u^* = \ln y^*(\theta)$ for the threshold in logs.

Firms enter the population at rate η, and the log dividend of an entrant is drawn from a density g with support in $[u^*, \infty)$. The support restriction reflects how firms enter the data: every firm in our sample raised a first round, so it was funded at entry, and a firm is funded only above the threshold. We place no other restriction on g, since the results below do not depend on it.

Let $f(u)$ be the stationary distribution of firms over the log dividend, so that $f(u)\,\mathrm{d}u$ is the mass of firms with log dividend in $[u, u + \mathrm{d}u)$. The stationary distribution satisfies the usual Kolmogorov forward equation

$$0 = \underbrace{-m f'(u) + \frac{\sigma^2}{2} f''(u)}_{\text{diffusion}} \underbrace{- \phi f(u)}_{\text{payout}} \underbrace{- \lambda \mathbb{1}_{\{u<u^*\}} f(u)}_{\text{failure}} + \underbrace{\eta g(u)}_{\text{entry}} \tag{11}$$

The first two terms are the net flow of firms into u from the drift and the volatility of the log dividend. The third is the flow of firms that leave at u because the payout arrives, at rate ϕ, wherever they are. The fourth is the flow that leaves because a financing need arrives while the dividend is below the threshold. Above the threshold a financing need is funded and the firm stays where it is, so financing needs do not appear there. The last term is the flow of entrants at u.

Since we are interested in the fraction of failures that bargaining power causes, and failures occur only below the threshold, we can focus on solving Eq. (11) in that region. Below u^* there is no entry, so the equation is homogeneous, and its solution depends on the entry distribution only through a constant that does not matter for the share of failures.

Proposition 4 (Inefficient failures)**.** *In the stationary state, the share of failures that occur at*

dividends the efficient benchmark would fund is

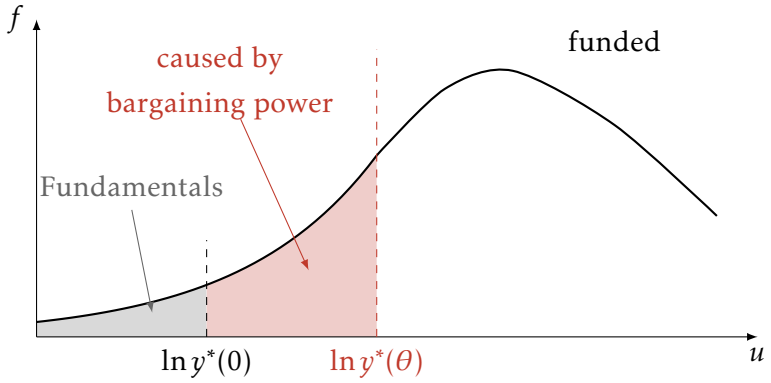
$$1 - \left(\frac{y^*(0)}{y^*(\theta)}\right)^{\kappa}, \qquad \kappa = \frac{m + \sqrt{m^2 + 2\sigma^2(\phi + \lambda)}}{\sigma^2} > 0 \tag{12}$$

It does not depend on the entry rate η, on the entry distribution g, or on the slope α of the financing need.

Proof: See Appendix A.8.

Figure 4 shows the stationary density and the two groups of failures. Below $\ln y^*(\theta)$ every firm there fails when its next financing need arrives. The firms between $\ln y^*(0)$ and $\ln y^*(\theta)$ are worth funding and fail because of bargaining power, and Eq. (12) is the share of the failure flow that comes from this region. The share depends on the parameters only through κ, which is determined by $(\mu, \sigma, \phi, \lambda)$, and through the ratio of the two thresholds.

Figure 4: Inefficient failures



Proposition 4 counts the firms that removing bargaining power would save at the moment it is removed. A firm in the region $[y^*(0), y^*(\theta))$ that is hit by a financing need would be funded in the absence of bargaining power, so that failure would not happen. Not all of these firms would reach the payout, however. Some of them would later drift below $y^*(0)$ and be hit by a financing need while there, so they would fail even without bargaining power, only later. We therefore ask a second question: of the firms that removing bargaining power saves, what fraction survives to the payout?

To answer this question, we compare the failure flow in the stationary state with bargaining power to the failure flow in the stationary state without it. We hold the entry process fixed, so firms enter at the same rate η and with the same density g, whose support remains $[\ln y^*(\theta), \infty)$, so that the comparison isolates the effect of the threshold.

Proposition 5 (Counterfactual failure flow)**.** *In the stationary state, removing bargaining power*

while holding the entry process fixed reduces the failure flow by:

$$1 - \left(\frac{y^*(0)}{y^*(\theta)}\right)^{\zeta}, \qquad \zeta = \frac{m + \sqrt{m^2 + 2\sigma^2\phi}}{\sigma^2} > 0 \tag{13}$$

It does not depend on the entry rate η, on the entry distribution g, or on the slope α of the financing need.

Proof: See Appendix A.9.

The exponent ζ is the exponent κ of Proposition 4 with ϕ in place of $\phi + \lambda$, so $\zeta < \kappa$ and the share in Eq. (13) is smaller than the share in Eq. (12), as it should. The difference represents the firms that bargaining power kills at one financing need and that would have failed at a later one anyway even without bargaining power. The ratio of the two shares is the fraction of the firms saved at the moment of failure that go on to the payout. Notice that λ enters Eq. (13) only through the thresholds. The arrival rate of financing needs governs the probability that a firm fails once its dividend has reached the threshold, and that probability is the same under both thresholds, so it cancels in the ratio. What differs is how far an entrant's dividend has to drift before it reaches the threshold, and whether it gets there before the payout arrives, which is what ζ summarizes.

4 Data

This section describes the data and maps them to the model. We build a panel of venture-backed firms from PitchBook, reduce their deals to financing events that correspond to the model's rounds, reconstruct how each firm's history ends, link the firms to employment records from Revelio Labs, which we use as a proxy for a firm's fundamentals between rounds, and select the sample on which the model is estimated. The last part of the section states the choices that the mapping requires, in particular how a round is classified as inside or market and how failures are dated.

4.1. Source and Sample

Our data come from PitchBook, a commercial database of private-market transactions that records, for each deal, the company, the investors, the amount raised, the valuation and the terms, and for each company its headquarters, its business status and its ownership status. We use the full export dated October 31, 2025, and everything measured in time, the observation windows, the failure dates and the sample cutoffs, is measured against that date.

We start from the 2.7 million deals in the export and keep venture capital deals that were completed and have a valid date. We then apply two filters at the company level. First, the

company is headquartered in the United States. Second, its first venture capital deal is on or after January 1, 2005, so that every firm in the panel is observed from its first round and its round history is complete. Within these companies, the financing rounds are the primary financings, seed, angel, accelerator, early and later stage venture rounds, growth rounds and convertible notes. Debt, secondary transactions, grants and joint ventures are not financing rounds and are dropped. Acquisitions and buyouts are not financing rounds either, since the firm raises no equity in them, but they are how many firms leave the panel, and we use them below as exits, taken from the full list of deals. Finally, we require the firm's first priced round to be on or before October 31, 2018, seven years before the end of the data, so that every firm has enough time in the sample to exit or fail.[10] The panel contains 41,722 firms whose first priced round falls between January 2005 and October 2018, observed through October 2025.

4.2. Financing Events

A round in the model is a single event at which the firm raises capital and sells equity. PitchBook records deals, which do not map one to one into rounds, for two reasons. First, a round sometimes closes in several tranches, recorded as separate deals. We collapse the closes of the same stock series within ninety days of the first close into one financing event, summing the amounts and taking the valuation, the series and the price per share from the last close.[11] Second, some financings are bridges rather than rounds. We classify an event as unpriced if it involves a convertible note or a SAFE and shows no evidence of pricing, such as an issue of preferred or common stock, a series label or a price per share. Unpriced events are bridges to the next round, not rounds in the model's sense, so they stay in the panel but are not counted as rounds. Events before a firm's first priced round are dropped, so that the first round of every firm is a priced equity round.

For each priced round we build the following variables the estimation uses. The capital raised is the equity invested in the event, from PitchBook's total invested equity where reported and otherwise from the deal size net of any debt component. The stake sold is the percentage of the firm acquired by all the investors who invested in the round, from filings where reported and otherwise the ratio of the equity raised to the post-money valuation, and it is observed for 85% of the priced rounds in the estimation sample.[12] The post-money valuation is observed for 84% of them. The investors in a round are the records with status new, follow-on or active investor, from which we count the investors and the new investors in each event. PitchBook also flags, for each investor record, whether that investor led the round.

[10] The cutoff is defined from the end of the data rather than by calendar year. Without it the most recent cohorts would be observed for too short a time to fail, and their presence would understate the failure rate. The choice of seven years for the cutoff follows Ewens, Rhodes-Kropf and Strebulaev (2016).

[11] The rule applies to 37 deals, 0.07% of the financing events in the estimation sample.

[12] Where both measures exist they agree, and the ratio fallback covers a small selected set of rounds (3%).

The type of a round follows from its lead investor, for the reasons given in Section 5. A round is a market round if a new investor leads it and an inside round if every recorded lead is a returning investor. PitchBook records a lead for 38% of the priced rounds in the estimation sample, and rounds without a recorded lead are left untyped. Only the moments that involve the type of a round, and the probability p, use this field. A firm with an untyped round contributes that round to every other moment and stays in the sample.[13] The first round of a firm is a market round in the model, and 99% of the first rounds with a recorded lead are led by a new investor in the data, so the moments that involve the type of a round, and p, are computed on rounds two and later.[14]

4.3. Terminal Events

Each firm's history ends in one of three ways, an exit, a failure, or censoring at the end of the data. Exits take precedence over failures, that is, if a firm is first acquired and later fails, we consider it as having a successful exit. An exit is the firm's first initial public offering, merger or acquisition, or buyout dated on or after its entry into the panel, taken from the full list of deals. Firms with no such deal whose ownership status is acquired, publicly held or in registration are given an exit at the date of that status, without a valuation. The exit value is the post-money valuation of the exit deal. For an initial public offering it is the offer price times the shares outstanding after the issue. For an acquisition or a buyout it is the post-money valuation where reported and the deal size otherwise, and the two coincide where both exist.

A firm that has not exited is classified as failed by either of two rules. The first is PitchBook's own record, a business status of out of business or bankruptcy, or an ownership status of out of business. The second is a twelve-year rule, in the spirit of Ewens, Rhodes-Kropf and Strebulaev (2016), who treat a firm that raised its first round early enough and has not exited by the end of their data as a failed investment. A firm whose first priced round is at least twelve years before the end of the data, on or before October 31, 2013, and that has not exited, is classified as failed. The failure date is the firm's last recorded activity of any kind plus one year, capped at the end of the data for firms failed only by the twelve-year rule and active within the last year.[15] A firm that has not exited or failed by the end of the data is censored at that date.

[13] Requiring a recorded lead on every round would keep 13% of the firms and select short-lived firms with good outcomes. Rounds with a recorded lead are the larger financings, raising about three times as much and selling a stake 4 percentage points higher than untyped rounds of the same number, year and sector, so the bargaining power is measured on the better-documented rounds, and both types of round face the same reporting filter.

[14] We follow the same convention as in Ewens, Rhodes-Kropf and Strebulaev (2016).

[15] In a later version of the paper we will use the employment records of Section 4.4 as an additional rule to classify a firm as failed among those falling under the twelve-year rule.

4.4. Employment

We complement PitchBook with employment data from Revelio Labs, accessed through WRDS. Revelio builds firm-level workforce records from the online professional profiles of individual workers, mostly on LinkedIn. For each worker it records every job spell, with the firm and the start and end dates, together with sampling weights that correct for the incomplete coverage of the workforce by online profiles. We aggregate the weighted spells into a monthly headcount for every firm, counting a worker as employed from the month a spell starts until the month it ends.

We link PitchBook companies to Revelio firms in four steps, in decreasing order of reliability. The first uses the LinkedIn company page recorded in both datasets, which identifies a company uniquely. The second, for the companies still unmatched, uses the website domain when it identifies a single Revelio firm. The third uses the website domain together with the company name when a domain is shared by a few firms. The fourth, for companies with no usable website, uses the company name and the state of the headquarters when exactly one Revelio firm matches both. Ambiguous cases are left unmatched. Of the 16,521 firms in the estimation sample, 90% are matched, 60% through the LinkedIn page and 25% through the website domain. Where PitchBook reports a headcount, the Revelio headcount in the same month is close to it, with a median ratio of 1.06.

We use the headcount as a proxy for the fundamentals of a firm between rounds. For each priced round, headcount growth is the log change in the headcount between the month of the previous priced round and the month of the round, with a floor of one employee. It is available for 7,875 of the 9,085 rounds used to estimate new investors' bargaining power in Section 5.1.

4.5. The Estimation Sample

The SMM part of the estimation simulates one model firm for each firm in the data, paired with that firm's stake at entry and its observation window. It therefore requires the stake sold in the first round to be observed, which defines the estimation sample, or entry pool. Of the 41,722 firms in the panel, 16,521 have an observed first-round stake in $(0, 1]$. Six in ten firms drop at this step because PitchBook does not record the share sold in their first round, and since disclosure improves over time the sample leans toward the later cohorts within the 2005 to 2018 window. Table 5 in Appendix B reports summary statistics for the firms in the sample and for their priced rounds.

Table 1 summarizes the mapping between the model and the data. A financing need is a priced round. The stake is the share of the firm sold in the round. The payout is an exit, and the dividend at the payout is the exit valuation. A failure is a firm that fails as defined above,

which in the model is a financing need that arrives while the dividend is below the threshold.

Table 1: Mapping the model to the data

Model	Data
Financing need, rate λ	Priced equity round
Inside round, market round	Round led by a returning investor, by a new investor
Stake s_i	Share of the firm sold in the round
Payout, rate ϕ	Initial public offering, acquisition or buyout
Dividend at the payout y_{τ_ϕ}	Valuation at the exit
Failure, $y < y^*$ at a financing need	Failure by status or by the twelve-year rule

5 Estimation

The model has nine parameters. Two describe the dividend, its drift μ and its volatility σ. Two describe the events in the firm's life, the arrival rate λ of financing needs and the arrival rate ϕ of the payout. Two describe the financing need, its fixed component x_0 and its slope α. Two describe the market rounds, the probability p that a round is a market round and new investors' bargaining power θ. The last is the discount rate ρ. Of these, x_0 is a normalization. The model is homogeneous of degree one in the dividend and the financing need, so scaling both by a common factor scales the equity value and the threshold by the same factor, and setting $x_0 = 1$ fixes the unit in which dividends are measured.

We estimate the remaining eight parameters in four steps, ordered so that each step uses only what the previous ones deliver. First, two parameters need no estimation. The probability of a market round, p, is the share of rounds in the data that a new investor leads, and the discount rate ρ comes from the literature. Second, new investors' bargaining power θ is estimated from the difference between the stakes sold at market and at inside rounds, which the model ties to θ alone. Third, the two arrival rates, ϕ and λ, are estimated by maximum likelihood on the timing of rounds, payouts and failures, which the model ties to the two Poisson clocks. Fourth, the drift and the volatility of the dividend, μ and σ, are estimated by the simulated method of moments, since the dividend is never observed and the moments that identify them depend on the whole path of the firm. The remainder of this section sets p and ρ. The three subsections that follow estimate new investors' bargaining power, the two arrival rates, and the drift and the volatility of the dividend.

Investors in the model are risk neutral and discount at rate ρ, so ρ is the expected return that investors require to hold venture equity between rounds. We take it from Burguet, Maréchal and Mermoud (2024), who revisit Cochrane (2005) on nearly 120,000 venture

capital valuations from 2010 to 2022, correcting for the selection of firms into observed valuations. They estimate an annualized expected return on venture capital investments of around 38%, so we set $\rho = 0.32$.[16]

The probability of a market round is the share of priced rounds two and later in the data at which a new lead investor enters. The first round is left out because the first round of every firm is a market round in the model (Section 4.2). In the data this share is 76%, so $p = 0.76$. We classify rounds by the lead investor rather than by any new investor for two reasons. The lead investor usually writes the largest check in the round. The lead investor also sets the terms of the deal, the valuation and the stake, and every other participant in the round invests on those terms. A round led by an investor already in the firm is priced by the owners and corresponds to an inside round in the model, whereas a round led by an outsider is priced by a newcomer who bargains with the owners and corresponds to a market round.

5.1. Measuring Bargaining Power

The model assumes that a market round is priced differently from an inside round. At an inside round the owners put in the capital pro rata, which is equivalent to issuing new shares at the equity value (Appendix A.1), so the stake sold in the round is the share of the firm that the new capital represents[17]

$$s^I(y) = \frac{x(y)}{W(y)}$$

At a market round, by Eq. (3), the new investor receives

$$s^M(y) = \theta + (1-\theta)\,\frac{x(y)}{W(y)}$$

At the same dividend y, and therefore the same state $x(y)/W(y)$, the owners keep after a market round a fraction $1-\theta$ of the share an inside round would leave them

$$\underbrace{1 - s^M(y)}_{\text{kept after a market round}} = (1-\theta)\ \underbrace{\left(1 - s^I(y)\right)}_{\text{kept after an inside round}}$$

[16] Their estimate is an arithmetic annual return, whereas ρ is continuously compounded, since the HJB equates ρ to the expected instantaneous return on the equity. The continuously compounded rate that delivers a 38% expected return over a year is $\rho = \ln(1.38) = 0.32$.

[17] Admati and Pfleiderer (1994) show that an inside investor who holds a fixed fraction of the firm and finances the same fraction of every later round has no incentive to misprice those rounds, which is the sense in which an inside round is priced at the equity value.

For round k of firm i, the previous expression can be written as:

$$\log(1 - s_{ik}) = \log(1 - \theta) M_{ik} + \log(1 - x_{ik}/W_{ik}) \tag{14}$$

where s_{ik} is the stake sold in the round, M_{ik} equals one if the round is a market round and zero if it is an inside round, x_{ik} is the capital raised in that round, and W_{ik} is the equity value once the round is funded. Equation (14) motivates the following regression

$$\log(1 - s_{ik}) = \beta M_{ik} + \Gamma' X_{ik} + \gamma_{r(ik)} + \delta_{t(ik)\times c(i)} + \chi_i + \varepsilon_{ik} \tag{15}$$

which we estimate on the priced rounds two and later of the estimation sample with a recorded lead investor and an observed stake, restricted to the firms with at least two such rounds, so that every column, including the one with firm fixed effects, is estimated on the same rounds. The dependent variable is the log of the share of the firm that the owners retain after the round, that is, one minus the stake sold. The coefficient of interest is β, and by Eq. (14) $\beta = \log(1 - \theta)$, so new investors' bargaining power is recovered as $\theta = 1 - e^{\beta}$.

The remaining terms stand in for the unobserved state $\log(1 - x_{ik}/W_{ik})$.[18] $\gamma_{r(ik)}$ is a fixed effect for the round number $r(ik)$. The state also varies with market conditions, so $\delta_{t(ik)\times c(i)}$ is a fixed effect for the calendar year of the round $t(ik)$ interacted with the firm's industry $c(i)$, PitchBook's industry group, of which there are 41. χ_i is a firm fixed effect, which compares the market and the inside rounds of the same firm.

The vector X_{ik} contains three controls for the state of the firm at the round. The first is the log of the capital raised, which enters the state directly and compares a market round to an inside round of the same size. The second is the log of the post-money valuation of the firm's previous priced round, the last valuation observed before the round. The third is the growth in the firm's headcount since the previous priced round, from the employment records of Section 4.4, which captures how the firm's fundamentals have evolved since the previous round.

Table 2 reports the estimates. Columns (1) to (3) control for the capital raised and the previous post-money valuation, and add the fixed effects one set at a time. Column (1) has none, column (2) adds the round-number and the year-by-industry fixed effects, and column (3) adds the firm fixed effect. Column (4) estimates column (3) on the rounds for which headcount growth is observed, and column (5) adds headcount growth, so the change from column (3) to column (4) reflects only the sample and the change from column (4) to column (5) only the additional control.

[18] Note that W_{ik} is the equity value at the time of round k, which is not the same as the post-money valuation in PitchBook. PitchBook's post-money valuation is calculated as x_{ik}/s_{ik}, which equals the equity value only at inside rounds, or when new investors have no bargaining power.

Across the specifications of Table 2, the estimate is small, between 0.3% and 2.1%. New investors capture only a small share of the surplus of a market round, 1.85% in column (5), which we take as the baseline because it controls for every fixed effect and every proxy for the state of the firm. Headcount growth barely moves the estimate, from 1.93% in column (4) to 1.85% in column (5), so the baseline does not depend on the employment records.

Market rounds and inside rounds differ not only in whether the lead investor is new to the firm but also in who the lead investor is. New lead investors may differ from the owners who lead inside rounds, for example in their reputation or in the services they bring, and an investor who is worth more to the firm may obtain a larger stake for the same capital for reasons other than bargaining power.[19] The gap in Table 2 would then reflect who leads the round rather than whether the lead is new to the firm. Column (2) of Table 8 in Appendix C.5 adds a fixed effect for the lead investor, so that the estimate compares rounds led by the same investor, as a new investor at some firms and as a returning one at others. The estimate is 1.89%. On the same rounds the baseline specification gives 2.14%, so the identity of the lead accounts for a small part of the gap, and the estimate remains close to the baseline.

The stake sold is not the only term of a round. A lead investor can also obtain different liquidation preferences, participation in the proceeds beyond the preference, or full-ratchet protection against later dilution, and each of these terms changes what a given stake is worth.[20] If market and inside rounds differed in these terms, the stake sold would misstate the difference in their prices. Column (3) of Table 8 adds indicators for the three terms, on the rounds for which they are observed. The estimate is 1.73%, the same as the baseline specification on those rounds. Differences in terms between market and inside rounds therefore do not explain the gap.

New investors may also know less about the firm than its owners. An investor who enters a firm for the first time has not observed its development, and an investor who is less informed than the owners faces adverse selection. It would then demand a larger stake to protect itself, and the gap between market and inside rounds would reflect asymmetric information rather than bargaining power. The asymmetry should be largest for young firms, which have little track record that an outsider can verify, and should shrink as the firm ages. We test this by

[19] Hsu (2004) compares competing offers made to the same start-ups at their first professional round and finds that offers from venture capitalists with a high reputation are three times more likely to be accepted and acquire equity at a 10 to 14% discount.

[20] Gornall and Strebulaev (2020) value the share classes of 135 US unicorns from the terms in their legal filings and find that the protections given to recent investors, such as seniority, return guarantees at an initial public offering or vetoes over a down-round offering, make their shares worth more than the common shares, so that reported post-money valuations average 48% above fair value.

letting the effect of a market round vary with the age of the firm at the round

$$\log(1 - s_{ik}) = \sum_{A} \beta_A M_{ik} \mathbb{1}_{\{a_{ik} \in A\}} + \Gamma' X_{ik} + \gamma_{r(ik)} + \delta_{t(ik)\times c(i)} + \chi_i + \varepsilon_{ik}$$

where a_{ik} is the age of firm i at round k, in years since its founding, the bins A are 0 to 2, 3 to 4, 5 to 6, 7 to 9, and 10 or more years, and the bargaining power in bin A is $\theta_A = 1 - e^{\beta_A}$. If the gap reflected asymmetric information, the estimate would be largest for the youngest firms and fall with age. Figure 5 in Appendix C.6 shows that it does not. The estimate for firms at most two years old is 0.9%, the lowest of the five, the estimates are not statistically different from one another.

Finally, we consider other definitions of a market round. Column (3) of Table 7 in Appendix C.4 classifies a round as a market round when at least one investor in it is new to the firm, whether or not that investor leads. The estimate is 1.53%, smaller than the baseline, as expected if a round that a new investor joins but does not lead is priced by the returning investors who lead it.

Table 2: New investors' bargaining power from the pricing of market and inside rounds

	(1)	(2)	(3)	(4)	(5)
Market round	−0.0032	−0.0100***	−0.0215***	−0.0195***	−0.0186***
	(0.0037)	(0.0036)	(0.0044)	(0.0045)	(0.0044)
log amount raised	−0.0958***	−0.0907***	−0.0952***	−0.0949***	−0.1013***
	(0.0031)	(0.0030)	(0.0042)	(0.0042)	(0.0042)
log previous post-money	0.1101***	0.1118***	0.0630***	0.0651***	0.0666***
	(0.0024)	(0.0027)	(0.0041)	(0.0042)	(0.0042)
Headcount growth					0.0306***
					(0.0052)
Implied θ (%)	0.32	1.00	2.13	1.93	1.85
	(0.37)	(0.36)	(0.43)	(0.44)	(0.44)
Round-number FE		✓	✓	✓	✓
Year × Industry FE		✓	✓	✓	✓
Company FE			✓	✓	✓
Sample	All	All	All	Revelio matched	Revelio matched
Observations	8,509	8,509	8,509	7,875	7,875
R^2	0.310	0.435	0.714	0.721	0.724

OLS estimates of $\log(1 - s_{ik}) = \beta M_{ik} + \Gamma' X_{ik} + \gamma_{r(ik)} + \delta_{t(ik)\times c(i)} + \chi_i + \varepsilon_{ik}$ on priced rounds $k \geq 2$ of company i with a recorded lead investor, an observed stake s_{ik} and a reported raise x_{ik}, for companies with at least two such rounds. M_{ik} indicates a market round (the lead investor is new to the company); the controls X_{ik} are the log equity raised, the log post-money valuation of the company's previous priced round and, in column (5), the log change in its headcount between that round and this one (Revelio Labs). $\gamma_{r(ik)}$, $\delta_{t(ik)\times c(i)}$ and χ_i are round-number, year × industry (PitchBook industry group, 41 groups) and company fixed effects. Columns (1)–(3) use the same rounds, those with the previous post-money; columns (4) and (5) use the rounds also matched to headcount growth, so that column (4) repeats column (3) on them. Column (5) is the baseline. Implied $\theta = 1 - e^{\beta}$ in percent, with delta-method standard errors. Standard errors clustered by company in parentheses; * $p < 0.10$, ** $p < 0.05$, *** $p < 0.01$.

5.2. Maximum Likelihood

The two arrival rates govern the timing of everything the panel records. The dividend determines whether a round is funded, not when a financing need arrives, and it plays no role in when the payout arrives. We can therefore estimate ϕ and λ from dates alone.

For each firm i the panel records the dates of its K_i rounds beyond entry, the years T_i during which the firm is at risk, that is, from its first round to its exit, its failure or the end of the sample, and how its history ends, with an exit, a failure or censoring. Entry is conditioned on, not modeled, since every firm in the sample raised a first round. Let N_E be the number of firms observed exiting and N_F the number observed failing. The maximum likelihood estimators are the two hazard rates

$$\hat{\phi} = \frac{N_E}{\sum_i T_i}, \qquad \hat{\lambda} = \frac{\sum_i K_i + N_F}{\sum_i T_i} \tag{16}$$

The estimator of ϕ counts exits per year at risk. The estimator of λ counts financing needs per year at risk, and in the model a financing need has one of two outcomes. If the dividend is above the threshold the round is funded and the panel records a round, and if it is below, the round is not funded and the panel records a failure. Both are arrivals of the financing clock, so both are counted, and a firm's failure is its last financing need. Leaving failures out would treat a firm that fails as one whose last financing need never happened, and would bias $\hat{\lambda}$ downward.

The two estimators require no correction for the selection of firms into failure. The log-likelihood of a history is the sum of a term in ϕ, a term in λ, and a term that collects every condition on the path of the dividend, that funded rounds occurred above the threshold and failures below it. The last term depends on the dividend parameters and on the threshold but not on ϕ or λ, because the clocks are independent of the dividend, so it drops out of the first-order conditions for the arrival rates.[21] The estimates of ϕ and λ are reported in Table 3.

5.3. Simulated Method of Moments

The remaining parameters are the drift and the volatility of the dividend, μ and σ. For now we set $\alpha = 0$, so that every round costs x_0[22]. The dividend is never observed, so neither parameter has a closed-form estimator, and we estimate them by the simulated method of moments. Given a candidate $\vartheta = (\mu, \sigma)$ and the parameters already in hand, we solve for the threshold, simulate a panel of firms with the same follow-up windows as the data, compute on the simulated panel the same two moments that we compute on the data, and search for the candidate that brings the two closest. The details of the simulation, the initial condition of the simulated firms, the treatment of censoring, and the computation of the moments are in Appendix C.

The two moments come from the firms that reach the payout, and they exploit a property of the model. The payout clock is independent of the dividend, so the date at which a firm's value is realized carries no information about that value, and conditional on the time T from entry to the payout, the log dividend at the payout is normal with mean $\ln y_0 + mT$ and variance $\sigma^2 T$, where $m = \mu - \sigma^2/2$. In a panel where every firm reaches the payout, a regression of the log exit value on T therefore recovers m as its slope, and a regression of its squared residuals on T recovers σ^2. For each firm that reaches the payout, let lr_i be the log of the exit value relative to the capital raised at entry, and T_i the years from the first round to the payout. We run the two regressions

$$lr_i = a + b\,T_i + e_i, \qquad e_i^2 = c + d\,T_i + u_i \tag{17}$$

[21] This is also why μ and σ cannot be estimated from dates. They enter the likelihood only through the conditions on the dividend path, which the simulated method of moments handles next.

[22] In a later version of this paper, α will be jointly estimated with μ and σ in the SMM part.

and use the two slopes.

(i) The slope b is the average growth rate of the log dividend among the firms that reach the payout, which under the geometric Brownian motion is $m = \mu - \sigma^2/2$, up to the selection of the firms that survive to the payout. Together with the volatility from the next moment, it pins down the drift μ.

(ii) The slope d is the rate at which the dispersion of the log dividend grows with time, which under the geometric Brownian motion is σ^2. It pins down the volatility.

Two moments identify two parameters, so the model is exactly identified.

In the data, only the firms that reach the payout have an exit value, and reaching the payout requires surviving every financing need along the way, which selects the paths that stayed above the threshold. The two slopes are therefore not m and σ^2 themselves, and we do not invert them. The simulated panel is subject to the same selection, since a simulated firm fails when its dividend is below the threshold at a financing need and only the simulated firms that reach the payout enter the regressions, so the simulated and the data slopes are computed on samples selected in the same way. The selection is what makes the slopes depend on the threshold, and through it on new investors' bargaining power and the arrival rates, as well as on m and σ.

We estimate ϑ by minimizing the sum of squared symmetric percentage deviations between the simulated and the data moments,

$$\hat{\vartheta} = \arg\min_{\vartheta} \sum_{j=1}^{2} \left[\frac{m_j(\vartheta) - \hat{m}_j}{\left(|m_j(\vartheta)| + |\hat{m}_j| \right)/2} \right]^2 \tag{18}$$

where $m_j(\vartheta)$ is the j-th moment on the simulated panel and $\hat{m}_j$ its counterpart in the data. The deviations are scale free, so the two slopes, which differ in magnitude, contribute equally. With two moments and two parameters the weighting does not matter at the solution, where both moments are matched.

Table 3 shows the parameters, their values and where each comes from.

Table 3: Parameters

	Parameter	Value	Source
x_0	Financing need, fixed component	1	Normalization
α	Financing need, slope	0	Assumption
ρ	Discount rate	0.32	Burguet, Maréchal and Mermoud (2024)
p	Probability of a market round	0.76	Share of rounds led by a new investor
θ	Bargaining power of new investors	1.85%	Table 2, column (5)
ϕ	Arrival rate of the payout	0.051	Maximum likelihood, Eq. (16)
λ	Arrival rate of financing needs	0.315	Maximum likelihood, Eq. (16)
μ	Drift of the dividend	0.270	SMM
σ	Volatility of the dividend	0.512	SMM

6 How Many Failures Does Bargaining Power Cause?

With the parameters in hand, the cost of new investors' bargaining power follows from Section 3 in closed form. Table 4 reports the two thresholds, the two exponents, and the two shares of Propositions 4 and 5 at the estimated parameters.

Table 4: The cost of bargaining power at the estimated parameters

Quantity	Value
Funding threshold $y^*(\theta)$	3.55
Efficient threshold $y^*(0)$	3.44
Ratio of thresholds $y^*(0)/y^*(\theta)$	0.97
Density exponent κ	2.28
Survival exponent ζ	1.35
Share of failures caused by bargaining power (%)	7.11
Share of failures prevented (%)	4.26

The cost of bargaining power is large relative to its size. New investors capture only 1.85% of the surplus of a market round, yet 7.1% of the failures in the stationary state occur at firms that the efficient benchmark funds, those whose dividend lies between $y^*(0)$ and $y^*(\theta)$ when a financing need arrives. Most of these firms would survive in a world where new investors have no bargaining power. Removing it lowers the failure flow by 4.3%, so six in ten of the firms that bargaining power kills would succeed, that is, would go public or be acquired, and the other four would fall below $y^*(0)$ and fail at a later financing need.

The effect is large because bargaining power compounds over every future market round. A new investor captures its share of the surplus only at its own round, but a firm expects to raise many market rounds before the payout, and the equity value today prices every one of them. Whoever funds the firm today bears the surplus that all future investors will capture, so bargaining power lowers the equity value at every dividend and raises the funding threshold from $y^*(0) = 3.44$ to $y^*(\theta) = 3.55$, in units of the financing need.

Figure 6 in Appendix D traces the two shares as θ varies from zero to 5%, holding every other parameter at its estimated value.[23] Both shares rise nearly in proportion to θ. Across the specifications of Table 2 with the firm fixed effect, whose estimates range from 1.85% to 2.13%, the share of failures that bargaining power causes ranges from 7.1% to 8.1%.

The cost of bargaining power also depends on the other parameters of the model, and Appendix D traces the two shares as each of them varies around its estimate, one at a time, holding the rest fixed. Both shares rise with the arrival rate of financing needs and with the probability of a market round, and fall with the arrival rate of the payout and with the discount rate.

7 Conclusion

This paper asks whether the bargaining power of future investors causes startups to fail and, if so, how many. We model staged equity financing as a dynamic game in which, at every round, either the current owners or a new investor decides whether to fund the firm, and a new investor bargains with the owners over the stake it receives. Inefficient failures arise instead whenever future investors have bargaining power. Bargaining power at the current round only splits the surplus between the owners and the new investor. Whoever funds the firm today, however, does not internalize the surplus that future investors will capture and treats it as a loss, so firms that are worth funding fail.

Our estimates show that new investors' bargaining power is small. Owners keep 1.85% less of the firm when a new investor leads a round than when one of the current owners leads it. Its cost, however, is large. Because it compounds over the many market rounds a firm expects to raise, it causes 7.1% of startup failures, and six in ten of those firms would otherwise go public or be acquired.

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A Omitted Proofs

A.1. Contributions at Inside Rounds

This appendix models an inside round as a game among the owners and shows that treating them as a single player, who funds the round if and only if the equity value covers its cost, is without loss of generality.

Consider an inside round at T_j and fix the funding sets (F, F^o) of every later round, so that the equity value once the round is funded, $W = W^{F,F^o}\left(y_{T_j}\right)$, is given, and write $x = x\left(y_{T_j}\right)$. The owners are indexed by $m = 1, \ldots, M$, and owner m holds a share $s^o_m > 0$ of the firm's equity, with $\sum_m s^o_m = 1$. Each owner chooses a contribution $c_m \geq 0$, and the round closes if and only if $\sum_m c_m = x$. If it closes, new shares are issued at the equity value, so owner m receives a share c_m/W of the firm and the shares held before the round are diluted by the factor $1 - x/W$. If it does not close, the contributions are returned, the firm goes bankrupt, and every owner receives zero.

In any profile that closes the round, the payoff of owner m is

$$W\Bigg[\underbrace{s^o_m\left(1-\frac{x}{W}\right)}_{\text{previous share after dilution}} + \underbrace{\frac{c_m}{W}}_{\text{new share}}\Bigg] - c_m = s^o_m\,(W - x)$$

which depends neither on its own contribution nor on the contributions of the other owners. How the owners divide the cost of the round therefore does not affect the payoff of any of them, and their payoffs sum to $W - x$, the payoff of a single player holding all of the equity. Pro rata contributions, $c_m = s^o_m x$, leave owner m with the share $s^o_m\,(1 - x/W) + s^o_m x/W = s^o_m$ it held before the round, which is the convention of Section 2.2.

Pro rata contributions are an equilibrium of this game if and only if $W \geq x$, the condition under which the owners fund in Eq. (5). Under pro rata contributions the round closes, and an owner who changes its contribution makes the sum differ from x, so the round fails and

the owner receives zero instead of $s_m^o(W-x)$. No owner gains from deviating if and only if $s_m^o(W-x) \geq 0$, that is, if and only if $W \geq x$. In the same way, the profile in which no owner contributes is an equilibrium if and only if $W \leq x$, since the only deviation that changes the outcome is to contribute x alone, which closes the round and pays $s_m^o(W-x)$.

A.2. The HJB Equation

This appendix states the HJB equation satisfied by the equity value under an arbitrary symmetric Markov profile and shows that it reduces to Eq. (6) in equilibrium. Applying Itô's lemma to Eq. (1), the equity value under a symmetric Markov profile (F, F^o) satisfies the following Hamilton-Jacobi-Bellman (HJB) equation

$$\begin{aligned}\rho W^{F,F^o}(y) = \mu y \frac{\mathrm{d}W^{F,F^o}}{\mathrm{d}y} + \frac{\sigma^2}{2} y^2 \frac{\mathrm{d}^2 W^{F,F^o}}{\mathrm{d}y^2} + \underbrace{\phi\left(y - W^{F,F^o}(y)\right)}_{\text{payout}} \\ - \lambda(1-p)\underbrace{\left[x(y)\,\mathbb{1}_{\{y\in F^o\}} + W^{F,F^o}(y)\,\mathbb{1}_{\{y\notin F^o\}}\right]}_{\text{inside round}} \\ + \lambda p \underbrace{\left[\left((1-\theta)\left(W^{F,F^o}(y) - x(y)\right) - W^{F,F^o}(y)\right)\mathbb{1}_{\{y\in F\}} - W^{F,F^o}(y)\,\mathbb{1}_{\{y\notin F\}}\right]}_{\text{market round}}\end{aligned} \tag{A.1}$$

The left side is the required return on the equity. On the right, the first two terms are the expected capital gain from the drift and the volatility of the dividend. The third term is the payout, at rate ϕ, at which the equity holders receive y and give up the equity value. The fourth term is an inside round, at rate $\lambda(1-p)$, at which the owners pay $x(y)$ if the dividend lies in F^o and lose everything otherwise. The fifth term is a market round, at rate λp. If the dividend lies in F, the equity holders keep the share $1-s(y)$ of the equity value, which equals $(1-\theta)\left(W^{F,F^o}(y) - x(y)\right)$ by Eq. (2), and if it does not, they lose everything.

In equilibrium $F = F^o = \{y : W(y) \geq x(y)\}$ by Lemma 1, and we write W for $W^{F,F}$. At a dividend in F, the inside and market round terms add up to

$$-\lambda(1-p)x(y) + \lambda p\left[(1-\theta)(W - x(y)) - W\right] = \lambda(1-p\theta)(W - x(y)) - \lambda W$$

and at a dividend outside F they add up to $-\lambda W$. Since $W \geq x(y)$ exactly on F, the two cases combine into $\lambda(1-p\theta)(W-x(y))^+ - \lambda W$, which gives Eq. (6).

A.3. Proof of Proposition 1

Proof. Fix a symmetric Markov equilibrium. By Lemma 1 its funding sets coincide, $F = F^o = \{y : W(y) \geq x(y)\}$, where $W = W^{F,F}$ is its equity value. We show that W is the fixed point of a contraction, so that any two equilibria have the same equity value and therefore the same funding set.

Let τ_λ be the arrival time of the first financing need, $\tau = \min\left\{\tau_\phi, \tau_\lambda\right\}$ the time of the first event, and $\mathbb{E}_y$ the expectation conditional on $y_0 = y$. At a financing need with dividend y, the equity holders keep all of the surplus at an inside round and a fraction $1-\theta$ of it at a market round, and the round is funded if and only if $y \in F$, so their expected payoff is

$$\psi(y) = (1-p)\,(W(y)-x(y))\,\mathbb{1}_{\{y\in F\}} + p\,(1-\theta)\,(W(y)-x(y))\,\mathbb{1}_{\{y\in F\}} = (1-p\theta)\,(W(y)-x(y))^+$$

where the last equality uses $F = \{y : W(y) \geq x(y)\}$. We can write $W(y)$ as the expected discounted payoff at the first event, which is either a payout or a financing need:

$$W(y) = \mathbb{E}_y\Big[e^{-\rho\tau}\left(y_\tau\,\mathbb{1}_{\{\tau_\phi<\tau_\lambda\}} + \psi(y_\tau)\,\mathbb{1}_{\{\tau_\lambda<\tau_\phi\}}\right)\Big]$$

The clocks τ_ϕ and τ_λ are independent exponentials with rates ϕ and λ, independent of the dividend. Conditioning on $\tau_\phi = s$ and $\tau_\lambda = t$ and using the law of iterated expectations,

$$\begin{aligned}
W(y) &= \int_0^\infty\!\!\int_0^\infty \phi e^{-\phi s}\,\lambda e^{-\lambda t}\left[e^{-\rho s}\,\mathbb{E}_y[y_s]\,\mathbb{1}_{\{s<t\}} + e^{-\rho t}\,\mathbb{E}_y[\psi(y_t)]\,\mathbb{1}_{\{t<s\}}\right]\mathrm{d}s\,\mathrm{d}t \\
&= \int_0^\infty \phi e^{-(\phi+\rho)s}\,\mathbb{E}_y[y_s]\left(\lambda\int_s^\infty e^{-\lambda t}\,\mathrm{d}t\right)\mathrm{d}s + \int_0^\infty \lambda e^{-(\lambda+\rho)t}\,\mathbb{E}_y[\psi(y_t)]\left(\int_t^\infty \phi e^{-\phi s}\,\mathrm{d}s\right)\mathrm{d}t \\
&= \int_0^\infty e^{-(\lambda+\rho+\phi)t}\,\mathbb{E}_y[\phi y_t + \lambda\psi(y_t)]\,\mathrm{d}t
\end{aligned}$$

which yields the fixed point equation:

$$W(y) = \mathbb{E}_y\left[\int_0^\infty e^{-(\lambda+\rho+\phi)t}\left[\phi y_t + \lambda\,(1-p\theta)\,(W(y_t)-x(y_t))^+\right]\mathrm{d}t\right] \tag{A.2}$$

Let $\mathcal{B}$ be the space of measurable functions $V : (0,\infty) \to \mathbb{R}$ with the following norm:

$$\|V\| = \sup_{y>0}\frac{|V(y)|}{1+y}$$

For $V \in \mathcal{B}$ define:

$$(TV)(y) = \mathbb{E}_y\left[\int_0^\infty e^{-(\lambda+\rho+\phi)t}\left[\phi y_t + \lambda\,(1-p\theta)\,(V(y_t)-x(y_t))^+\right]\mathrm{d}t\right]$$

Since $|V(y_t)| \le \|V\|(1+y_t)$ and $\mathbb{E}_y[y_t] = ye^{\mu t}$ with $\mu < \lambda+\rho+\phi$, TV has finite norm, so T maps $\mathcal{B}$ into itself, and Eq. (A.2) says that the equilibrium equity value is a fixed point of T.

For $V_1, V_2 \in \mathcal{B}$, the payout terms cancel and

$$(TV_1)(y) - (TV_2)(y) = \lambda(1-p\theta)\,\mathbb{E}_y\left[\int_0^\infty e^{-(\lambda+\rho+\phi)t}\left[(V_1(y_t)-x(y_t))^+ - (V_2(y_t)-x(y_t))^+\right]\mathrm{d}t\right]$$

For a, b, x we have: $|(a-x)^+ - (b-x)^+| \le |a-b|$. Taking absolute values and using $|V_1(y_t) - V_2(y_t)| \le \|V_1 - V_2\|(1+y_t)$,

$$\begin{aligned}
|(TV_1)(y) - (TV_2)(y)| &\le \lambda(1-p\theta)\|V_1-V_2\|\int_0^\infty e^{-(\lambda+\rho+\phi)t}\left(1+ye^{\mu t}\right)\mathrm{d}t \\
&= \lambda(1-p\theta)\|V_1-V_2\|\left(\frac{1}{\lambda+\rho+\phi} + \frac{y}{\lambda+\rho+\phi-\mu}\right) \\
&\le \frac{\lambda(1-p\theta)}{\min\{\lambda+\rho+\phi,\,\lambda+\rho+\phi-\mu\}}\|V_1-V_2\|(1+y)
\end{aligned}$$

Dividing by $1+y$ and taking the supremum over $y > 0$,

$$\|TV_1 - TV_2\| \le q\|V_1 - V_2\|, \qquad q = \frac{\lambda(1-p\theta)}{\min\{\lambda+\rho+\phi,\,\lambda+\rho+\phi-\mu\}}$$

If $\mu \ge 0$ the minimum is $\lambda+\rho+\phi-\mu$, and $q < 1$ if and only if $\lambda(1-p\theta) < \lambda+\rho+\phi-\mu$, that is, $\mu < \rho+\phi+\lambda p\theta$. If $\mu < 0$ the minimum is $\lambda+\rho+\phi$, and $q < 1$ if and only if $0 < \rho+\phi+\lambda p\theta$, which holds trivially. In both cases $q < 1$ under Assumption 1, so T is a contraction on $\mathcal{B}$ and has at most one fixed point.

Any two symmetric Markov equilibria therefore have the same equity value W, hence the same funding set $F = \{y : W(y) \ge x(y)\}$ and, by Eq. (3), the same stake at every market round, which is the statement of Proposition 1. □

A.4. Proof of Proposition 2

Proof. Throughout, W is the equilibrium equity value, which solves Eq. (6), and we write

$$R = \rho+\phi+\lambda p\theta-\mu, \qquad S = \rho+\phi+\lambda-\mu$$

Both are positive under Assumption 1, and $S - R = \lambda(1-p\theta)$. We first conjecture that $W(y) - x(y)$ is strictly increasing, solve Eq. (6) on each side of the threshold this conjecture implies, and pin down the threshold from the value of the equity there. The remaining steps verify the conjecture and establish the comparative statics.

The equity value on each side of the threshold. Suppose that $W(y) - x(y)$ is strictly increasing. Then $W(y) \geq x(y)$ if and only if $y \geq y^*$, where y^* solves $W(y^*) = x(y^*)$, and Eq. (6) is a linear second-order differential equation on each side of y^*.

In the unfunded region, $y < y^*$, the surplus term vanishes and Eq. (6) becomes

$$(\rho + \phi + \lambda) W = \mu y W' + \frac{\sigma^2}{2} y^2 W'' + \phi y$$

A linear particular solution $W = my$ requires $(\rho + \phi + \lambda) m = \mu m + \phi$, that is, $m_U = \phi / S$. The homogeneous equation has solutions y^γ, with γ a root of the characteristic equation

$$\frac{\sigma^2}{2}\gamma^2 + \left(\mu - \frac{\sigma^2}{2}\right)\gamma - (\rho + \phi + \lambda) = 0$$

The product of the roots is $-2(\rho + \phi + \lambda)/\sigma^2 < 0$, so one root is negative and the other positive, and since the left side equals $\mu - (\rho + \phi + \lambda) < 0$ at $\gamma = 1$, the positive root exceeds one. The negative root is ruled out, since if $\gamma < 0$, y^γ diverges as $y \to 0$, whereas the equity value is bounded near zero. Hence

$$W(y) = m_U y + B y^\gamma, \qquad \gamma = \frac{-\left(\mu - \frac{\sigma^2}{2}\right) + \sqrt{\left(\mu - \frac{\sigma^2}{2}\right)^2 + 2\sigma^2(\rho + \phi + \lambda)}}{\sigma^2} > 1$$

for some constant B.

In the funded region, $y \geq y^*$, the surplus is $W - \alpha y - x_0$ and Eq. (6) becomes

$$(\rho + \phi + \lambda p\theta) W = \mu y W' + \frac{\sigma^2}{2} y^2 W'' + (\phi - \alpha\lambda(1 - p\theta)) y - \lambda(1 - p\theta) x_0$$

A particular solution of the form $W = m_F y - c$ requires, matching the coefficients of y and the constants,

$$m_F = \frac{\phi - \alpha\lambda(1 - p\theta)}{R}, \qquad c = \frac{\lambda(1 - p\theta) x_0}{\rho + \phi + \lambda p\theta}$$

The slope m_F is positive under Assumptions 1 and 2, since $\phi > \alpha S \geq \alpha\lambda(1 - p\theta)$, where the first inequality is Assumption 2 and the second is $\alpha R \geq 0$. The homogeneous equation has solutions y^β, with β a root of

$$\frac{\sigma^2}{2}\beta^2 + \left(\mu - \frac{\sigma^2}{2}\right)\beta - (\rho + \phi + \lambda p\theta) = 0$$

As before, one root is negative and the other positive, and since the left side equals $\mu - (\rho + \phi + \lambda p\theta) < 0$ at $\beta = 1$ by Assumption 1, the positive root exceeds one. The positive root is ruled out since if $\beta > 1$, y^β grows faster than y, whereas the equity value grows at most

linearly. Hence

$$W(y) = m_F\, y - c + A\, y^{\beta}, \qquad \beta = \frac{-\left(\mu - \frac{\sigma^2}{2}\right) - \sqrt{\left(\mu - \frac{\sigma^2}{2}\right)^2 + 2\sigma^2\,(\rho + \phi + \lambda p \theta)}}{\sigma^2} < 0$$

for some constant A. Collecting the two regions,

$$W(y) = \begin{cases} m_F\, y - c + A\, y^{\beta} & y \geq y^* \\ m_U\, y + B\, y^{\gamma} & y < y^* \end{cases} \tag{A.3}$$

Neither β nor γ depends on α. Of the two, only β depends on θ, through $\lambda p \theta$.

The threshold. The constants A and B and the threshold y^* are pinned down by three conditions: W is continuous at y^*, W' is continuous at y^*, and the equity value equals the cost of the round at the threshold, $W(y^*) = \alpha y^* + x_0$.

The first two conditions give, after multiplying the second condition by y^*,

$$\begin{aligned} m_F\, y^* - c + A\,(y^*)^{\beta} &= m_U\, y^* + B\,(y^*)^{\gamma} \\ m_F\, y^* + \beta A\,(y^*)^{\beta} &= m_U\, y^* + \gamma B\,(y^*)^{\gamma} \end{aligned}$$

Writing $a = A\,(y^*)^{\beta}$ and $b = B\,(y^*)^{\gamma}$ for the two homogeneous terms at the threshold, this is a linear system in (a, b),

$$a - b = (m_U - m_F)\, y^* + c, \qquad \beta a - \gamma b = (m_U - m_F)\, y^*$$

with solution

$$a = \frac{(\gamma - 1)\,(m_U - m_F)\, y^* + \gamma c}{\gamma - \beta}, \qquad b = \frac{(\beta - 1)\,(m_U - m_F)\, y^* + \beta c}{\gamma - \beta}$$

The third condition is the definition of the threshold, $W(y^*) = \alpha y^* + x_0$. Evaluating W at y^* from the unfunded side of Eq. (A.3) and substituting b,

$$W(y^*) = m_U\, y^* + b = \frac{(\gamma - 1)\, m_U + (1 - \beta)\, m_F}{\gamma - \beta}\, y^* + \frac{\beta c}{\gamma - \beta}$$

Setting this equal to $\alpha y^* + x_0$ and solving for y^* gives

$$y^* = \frac{x_0\,(\gamma - \beta) - \beta c}{(\gamma - 1)\, m_U + (1 - \beta)\, m_F - \alpha\,(\gamma - \beta)} \tag{A.4}$$

The numerator is positive, since $\gamma > 1$, $\beta < 0$ and $c > 0$. We can simplify the denominator further. Note that:

$$(\gamma-1)\,m_U + (1-\beta)\,m_F - \alpha\,(\gamma-\beta) = \phi\left[\frac{\gamma-1}{S} + \frac{1-\beta}{R}\right] - \alpha\left[\gamma - \beta + \frac{(1-\beta)\,\lambda\,(1-p\theta)}{R}\right]$$

The two brackets are proportional. Using $\lambda(1-p\theta) = S - R$,

$$\gamma - \beta + \frac{(1-\beta)(S-R)}{R} = \frac{(\gamma-1)\,R + (1-\beta)\,S}{R} = \frac{S}{\phi}\,\phi\left[\frac{\gamma-1}{S} + \frac{1-\beta}{R}\right]$$

so the denominator of Eq. (A.4) equals $\phi\left[\frac{\gamma-1}{S} + \frac{1-\beta}{R}\right](1 - \alpha S/\phi)$, and

$$y^*(\theta) = \frac{\phi}{\phi - \alpha S}\,\tilde{y}^*(\theta), \qquad \tilde{y}^*(\theta) = \frac{x_0}{\phi} \cdot \frac{\gamma - \beta\,\dfrac{\rho+\phi+\lambda}{\rho+\phi+\lambda p\theta}}{\dfrac{\gamma-1}{S} + \dfrac{1-\beta}{R}} \tag{A.5}$$

where $\tilde{y}^*(\theta)$ is the threshold when $\alpha = 0$. The bracket in the denominator of $\tilde{y}^*$ is positive, so $\tilde{y}^*(\theta) > 0$ when $x_0 > 0$, and Assumption 2 is exactly the condition $\phi - \alpha S > 0$ that makes $y^*(\theta)$ positive and finite. The factor $\phi/(\phi - \alpha S)$ does not depend on θ, so the slope of the financing need scales the threshold by the same factor for every value of new investors' bargaining power.

$W(y) - \alpha y$ is strictly increasing below the threshold. We now verify the conjecture, starting with the unfunded region. By Eq. (A.3), for $y < y^*$,

$$W(y) - \alpha y - x_0 = (m_U - \alpha)\,y + B\,y^{\gamma} - x_0$$

The first term is strictly increasing, since $m_U - \alpha = (\phi - \alpha S)/S > 0$ by Assumption 2, and the second is nondecreasing if $B \geq 0$, since $\gamma > 1$. It remains to sign B. Evaluating the unfunded branch at the threshold, where $W(y^*) = \alpha y^* + x_0$,

$$B\,(y^*)^{\gamma} = x_0 - (m_U - \alpha)\,y^* = x_0 - m_U\,\tilde{y}^*(\theta)$$

where the second equality uses Eq. (A.5) and $m_U - \alpha = (\phi - \alpha S)/S$, so the right side does not depend on α. Substituting $\tilde{y}^*$ from Eq. (A.5) and using $m_U = \phi/S$,

$$x_0 - m_U\,\tilde{y}^*(\theta) = \frac{x_0}{\gamma - 1 + (1-\beta)\,\dfrac{S}{R}}\left[-1 + (1-\beta)\,\frac{S}{R} + \beta\,\frac{\rho+\phi+\lambda}{\rho+\phi+\lambda p\theta}\right]$$

Writing $S/R = 1 + \lambda(1-p\theta)/R$ and $(\rho+\phi+\lambda)/(\rho+\phi+\lambda p\theta) = 1 + \lambda(1-p\theta)/(\rho+\phi+\lambda p\theta)$, the constant terms in the bracket cancel and it equals

$$\lambda(1-p\theta)\left[\frac{1-\beta}{R} + \frac{\beta}{\rho+\phi+\lambda p\theta}\right] = \lambda(1-p\theta)\,\frac{\rho+\phi+\lambda p\theta-\mu\beta}{R(\rho+\phi+\lambda p\theta)}$$

The characteristic equation for β can be rearranged as $\frac{\sigma^2}{2}\beta(\beta-1) = \rho+\phi+\lambda p\theta-\mu\beta$, and the left side is positive because $\beta < 0$. Every factor is therefore positive, and

$$B(y^*)^\gamma = \frac{x_0\,\lambda(1-p\theta)}{\gamma - 1 + (1-\beta)\dfrac{S}{R}} \cdot \frac{\rho+\phi+\lambda p\theta-\mu\beta}{R(\rho+\phi+\lambda p\theta)} \geq 0 \tag{A.6}$$

with equality only if $p\theta = 1$. This gives us that $B \geq 0$. Hence $W(y) - \alpha y$ is strictly increasing on $(0, y^*)$, and since it equals x_0 at y^*, $W(y) < x(y)$ for every $y < y^*$, as conjectured.

$W(y) - \alpha y$ is strictly increasing above the threshold. By Eq. (A.3), for $y > y^*$,

$$\frac{\mathrm{d}}{\mathrm{d}y}[W(y)-\alpha y] = m_F - \alpha + A\beta\, y^{\beta-1}, \qquad m_F - \alpha = \frac{\phi - \alpha\lambda(1-p\theta) - \alpha R}{R} = \frac{\phi - \alpha S}{R} > 0$$

where the inequality is Assumption 2 again. If $A \leq 0$, the second term is nonnegative because $\beta < 0$, and the derivative is positive. If $A > 0$, the second term is negative, but

$$\frac{\mathrm{d}^2}{\mathrm{d}y^2}[W(y)-\alpha y] = A\beta(\beta-1)\,y^{\beta-2} > 0$$

since $\beta < 0$ and $\beta - 1 < 0$, so the derivative is increasing on (y^*, ∞). Its value as $y \downarrow y^*$ equals its value as $y \uparrow y^*$, because W' is continuous at y^*, and the latter is $m_U - \alpha + \gamma B(y^*)^{\gamma-1} > 0$ by the previous paragraph. Hence the derivative is positive on (y^*, ∞) in this case as well.

Together with the result below the threshold, $W(y) - \alpha y$ is strictly increasing on $(0, \infty)$, so

$$\{y : W(y) \geq x(y)\} = [y^*, \infty)$$

and the conjecture holds. Since T has exactly one fixed point, every symmetric Markov equilibrium has this equity value and this funding set, which proves the first part of Proposition 2.

Comparative statics: α. None of β, γ, R and S depends on α, so $\tilde{y}^*(\theta)$ in Eq. (A.5) does not depend on α either, and

$$\frac{\partial y^*(\theta)}{\partial \alpha} = \tilde{y}^*(\theta)\,\frac{\phi S}{(\phi - \alpha S)^2} > 0$$

on the range $\alpha < \phi/S$ allowed by Assumption 2. Thus, the threshold is increasing in α.

Simplifying $\tilde{y}^*$. The remaining parameters enter Eq. (A.5) through $\tilde{y}^*(\theta)$ and, in the case of λ, through S. It is convenient to write

$$\nu = \frac{\sigma^2}{2}, \qquad Q = \rho + \phi + \lambda = S + \mu, \qquad q = \rho + \phi + \lambda p\theta = R + \mu$$

so that the two characteristic equations are $\nu\gamma^2 + (\mu - \nu)\gamma - Q = 0$ and $\nu\beta^2 + (\mu - \nu)\beta - q = 0$, and $\tilde{y}^*(\theta) = \frac{x_0}{\phi}\left(\gamma - \beta\frac{Q}{q}\right)\Big/\left(\frac{\gamma-1}{Q-\mu} + \frac{1-\beta}{q-\mu}\right)$.

We have the following identities for any root z of $\nu z^2 + (\mu - \nu)z - r = 0$,

$$r - \mu = \nu z^2 + (\mu - \nu)z - \mu = (z-1)(\nu z + \mu), \qquad r = z[\nu(z-1) + \mu]$$

Applying the first identity to γ and β, the denominator of $\tilde{y}^*$ becomes

$$\frac{\gamma - 1}{Q - \mu} + \frac{1 - \beta}{q - \mu} = \frac{1}{\gamma\nu + \mu} - \frac{1}{\beta\nu + \mu} = \frac{(\beta - \gamma)\nu}{(\gamma\nu + \mu)(\beta\nu + \mu)}$$

and applying the second to Q and q, the numerator becomes

$$\gamma - \beta\frac{Q}{q} = \gamma\left[1 - \frac{\nu(\gamma - 1) + \mu}{\nu(\beta - 1) + \mu}\right] = \frac{\gamma\nu(\beta - \gamma)}{\nu(\beta - 1) + \mu}$$

Taking the ratio, the factor $(\beta - \gamma)\nu$ cancels and

$$\tilde{y}^*(\theta) = \frac{x_0}{\phi}\gamma\frac{(\mu + \gamma\nu)(\mu + \beta\nu)}{\mu + \nu(\beta - 1)} \tag{A.7}$$

First, $\mu + \gamma\nu > 0$, since $\gamma(\mu + \gamma\nu) = Q + \gamma\nu > 0$ and $\gamma > 0$. Second, $\mu + \beta\nu < 0$, since $R = q - \mu = (\beta - 1)(\beta\nu + \mu) > 0$ and $\beta - 1 < 0$. Third, $\mu + \nu(\beta - 1) = (\mu + \beta\nu) - \nu < 0$. The last two factors are negative, so $\tilde{y}^*(\theta) > 0$, as found above.

Comparative statics: θ and p. New investors' bargaining power and the probability of a market round enter Eq. (A.7) only through β, and β depends on them only through $q = \rho + \phi + \lambda p\theta$. Neither affects γ, ν or the factor $\phi/(\phi - \alpha S)$ in Eq. (A.5). Differentiating the factor that contains β,

$$\frac{\mathrm{d}}{\mathrm{d}\beta}\left[\frac{\mu + \beta\nu}{\mu + \nu(\beta - 1)}\right] = \frac{\nu[\mu + \nu(\beta - 1)] - \nu(\mu + \beta\nu)}{[\mu + \nu(\beta - 1)]^2} = \frac{-\nu^2}{[\mu + \nu(\beta - 1)]^2} < 0$$

so the threshold is decreasing in β. Differentiating the characteristic equation of β implicitly with respect to q,

$$\frac{\partial \beta}{\partial q} = \frac{1}{2\nu\beta + \mu - \nu} < 0$$

where the denominator is the slope of the convex quadratic $\nu z^2 + (\mu - \nu)z - q$ at its smaller root, which is negative. By the chain rule,

$$\frac{\partial y^*(\theta)}{\partial \theta} = \frac{\partial y^*(\theta)}{\partial \beta}\frac{\partial \beta}{\partial q}\lambda p, \qquad \frac{\partial y^*(\theta)}{\partial p} = \frac{\partial y^*(\theta)}{\partial \beta}\frac{\partial \beta}{\partial q}\lambda\theta$$

The product of the first two factors is positive, so the threshold is strictly increasing in θ if $p > 0$ and strictly increasing in p if $\theta > 0$.

Comparative statics: λ. The arrival rate of financing needs enters $y^*(\theta)$ in three ways: through Q in the characteristic equation of γ, through q in the characteristic equation of β, and through S in the factor $\phi/(\phi - \alpha S)$ of Eq. (A.5). Write Eq. (A.7) as

$$\tilde{y}^*(\theta) = \frac{x_0}{\phi} H G, \qquad H = \gamma(\mu + \gamma\nu), \qquad G = \frac{\mu + \beta\nu}{\mu + \nu(\beta - 1)}$$

where $H > 0$ and $G > 0$ by the signs established above. Differentiating,

$$\frac{\partial \tilde{y}^*(\theta)}{\partial \lambda} = \frac{x_0}{\phi}\left[\frac{\partial H}{\partial \gamma}\frac{\partial \gamma}{\partial Q} G + H \frac{\partial G}{\partial \beta}\frac{\partial \beta}{\partial q} p\theta\right]$$

since $\partial Q/\partial\lambda = 1$ and $\partial q/\partial\lambda = p\theta$. In the first term, $\partial H/\partial\gamma = \mu + 2\gamma\nu = \nu + \sqrt{(\mu - \nu)^2 + 4\nu Q} > 0$ from our previous expression for γ.

Now, $\partial\gamma/\partial Q = 1/(2\nu\gamma + \mu - \nu) > 0$, since the denominator is the slope of the convex quadratic $\nu z^2 + (\mu - \nu)z - Q$ evaluated at its larger root. The first term is therefore strictly positive.

In the second term, $\partial G/\partial\beta < 0$ and $\partial\beta/\partial q < 0$ from the previous paragraph, so the term is nonnegative, and it is zero only if $p\theta = 0$. Hence $\tilde{y}^*(\theta)$ is strictly increasing in λ. The factor $\phi/(\phi - \alpha S)$ is positive and nondecreasing in λ, since S is increasing in λ, so $y^*(\theta)$ is strictly increasing in λ.

Comparative statics: σ. Volatility enters $y^*(\theta)$ only through $\nu = \sigma^2/2$, and ν enters only through γ and β, since the factor $\phi/(\phi - \alpha S)$ does not involve σ. It therefore suffices to characterize the sign of $\partial\tilde{y}^*/\partial\nu$. Differentiating the two characteristic equations implicitly

with respect to ν,

$$\frac{\partial \gamma}{\partial \nu} = \frac{\gamma(1-\gamma)}{2\nu\gamma + \mu - \nu} < 0, \qquad \frac{\partial \beta}{\partial \nu} = \frac{\beta(1-\beta)}{2\nu\beta + \mu - \nu} > 0$$

The first is negative because $\gamma > 1$ and its denominator, the slope of the quadratic at its larger root, is positive. The second is positive because $\beta < 0$ makes its numerator negative and its denominator, the slope at the smaller root, is negative as well. Higher volatility pulls γ down and β up.

For this derivative it is convenient to return to the form of $\tilde{y}^*$ that precedes Eq. (A.7) and to separate the roots from the constants. Multiplying numerator and denominator by $q(Q-\mu)(q-\mu)$,

$$\tilde{y}^*(\theta) = \frac{x_0}{\phi} \frac{(Q-\mu)(q-\mu)}{q} \frac{N}{M}, \qquad N = \gamma q - \beta Q, \qquad M = (\gamma - 1)(q-\mu) + (1-\beta)(Q-\mu)$$

where only N and M depend on ν. By the quotient rule, $\partial(N/M)/\partial\nu = (N_\nu M - N M_\nu)/M^2$, with $N_\nu = \gamma_\nu q - \beta_\nu Q$ and $M_\nu = \gamma_\nu (q-\mu) - \beta_\nu (Q-\mu)$. Collecting the terms in γ_ν and in β_ν,

$$N_\nu M - N M_\nu = \gamma_\nu [qM - N(q-\mu)] - \beta_\nu [QM - N(Q-\mu)]$$

Each term simplifies to

$$qM - N(q-\mu) = (Q-q)(q-\beta\mu)$$
$$QM - N(Q-\mu) = (Q-q)(Q-\gamma\mu)$$

The characteristic equations give $q - \beta\mu = \nu\beta(\beta-1) > 0$ and $Q - \gamma\mu = \nu\gamma(\gamma-1) > 0$, and $Q - q = \lambda(1-p\theta)$. Therefore

$$\frac{\partial \tilde{y}^*(\theta)}{\partial \nu} = \frac{x_0}{\phi} \frac{(Q-\mu)(q-\mu)\lambda(1-p\theta)}{qM^2} \left[(q-\beta\mu)\frac{\partial \gamma}{\partial \nu} - (Q-\gamma\mu)\frac{\partial \beta}{\partial \nu} \right]$$

The term in the brackets is negative, since its first term is negative and its second is positive and enters with a minus sign, and the term multiplying it is positive when $p\theta < 1$. Hence $\tilde{y}^*(\theta)$, and with it $y^*(\theta)$, is strictly decreasing in ν, and so in σ, when $p\theta < 1$. When $p\theta = 1$, $Q = q$, γ and β are the two roots of the same quadratic, and $\tilde{y}^*$ reduces to $x_0 S/\phi = x_0/m_U$, which does not depend on σ. In that case the new investor takes the whole firm at every market round, so the owners value the firm as if it failed at its next financing need, which kills the option value. This completes the proof of Proposition 2. □

A.5. Neither Pessimism nor Optimism Is Self-Fulfilling

This appendix calculates the equity value under the two conjectures about future rounds that a self-fulfilling failure or a self-fulfilling success would require, that no future round will be funded and that every future round will be funded, and shows that the best response to each is a threshold. Throughout, Assumptions 1 and 2 hold and we write $S = \rho + \phi + \lambda - \mu$ and $R = \rho + \phi + \lambda p\theta - \mu$ as in Appendix A.4.

Suppose first that the equity holders expect every future round to not be funded. The equity then pays only if the payout arrives before the next financing need, so its value is

$$W^{\text{pess}}(y) = \mathbb{E}_y\left[e^{-\rho\tau_\phi}\, y_{\tau_\phi}\, \mathbb{1}_{\{\tau_\phi \le \tau_\lambda\}}\right]$$

The two clocks are independent exponentials with rates ϕ and λ, and $\mathbb{E}_y[y_s] = ye^{\mu s}$. Conditioning on $\tau_\phi = s$ and $\tau_\lambda = t$,

$$W^{\text{pess}}(y) = \int_0^\infty \phi e^{-\phi s}\, e^{-\rho s}\, y e^{\mu s} \left(\int_s^\infty \lambda e^{-\lambda t}\, \mathrm{d}t \right) \mathrm{d}s = \phi y \int_0^\infty e^{-Ss}\, \mathrm{d}s = \frac{\phi y}{S}$$

which is the slope m_U of the unfunded region in Appendix A.4. The active player compares this value with the cost of the round and funds if and only if $\phi y/S \ge x_0 + \alpha y$, that is,

$$y \ge y^{\text{pess}} = x_0 \frac{S}{\phi - \alpha S}$$

which is positive and finite under Assumption 2. The conjecture that no round is funded therefore induces the funding set $[y^{\text{pess}}, \infty)$ rather than the empty set, so it is not an equilibrium.

Suppose instead that the equity holders expect every future round to be funded, whatever the dividend. At an inside round they pay $x(y)$ and keep the firm, and at a market round they keep $(1 - s_i)\, W = (1 - \theta)(W - x(y))$ by Eq. (2), so the equity value solves

$$\begin{aligned} \rho W &= \mu y\, W' + \frac{\sigma^2}{2} y^2\, W'' + \phi\,(y - W) - \lambda\,(1 - p)\, x(y) - \lambda p\,[\theta W + (1 - \theta)\, x(y)] \\ &= \mu y\, W' + \frac{\sigma^2}{2} y^2\, W'' + \phi\,(y - W) - \lambda\,(1 - p\theta)\, x(y) - \lambda p\theta\, W \end{aligned}$$

on all of $(0, \infty)$. The homogeneous solutions are y^β with β a root of $\frac{\sigma^2}{2}\beta^2 + \left(\mu - \frac{\sigma^2}{2}\right)\beta - (\rho + \phi + \lambda p\theta) = 0$, one negative and one greater than one by Assumption 1. Both are ruled out, because the equation now holds on the whole positive line. The negative root makes y^β diverge as $y \to 0$, where the equity value is bounded, and the positive root makes it grow faster than y, whereas the equity value grows at most linearly. The equity value is therefore

the linear particular solution, and matching coefficients gives the one found for the funded region in Appendix A.4,

$$W^{\text{opt}}(y) = m_F\, y - c, \qquad m_F = \frac{\phi - \lambda\alpha\,(1 - p\theta)}{R}, \qquad c = \frac{\lambda x_0\,(1 - p\theta)}{\rho + \phi + \lambda p\theta}$$

The value is negative at low dividends, since the holders expect to pay for every round and to receive a payout that is worth little. The active player funds if and only if $m_F\, y - c \geq x_0 + \alpha y$. Using $m_F - \alpha = (\phi - \alpha S)/R$ and $x_0 + c = x_0\,(\rho + \phi + \lambda)/(\rho + \phi + \lambda p\theta)$,

$$y \geq y^{\text{opt}} = x_0\, \frac{\rho + \phi + \lambda}{\rho + \phi + \lambda p\theta} \cdot \frac{R}{\phi - \alpha S}$$

The conjecture that every round is funded induces the funding set $[y^{\text{opt}}, \infty)$ rather than $(0, \infty)$, so it is not an equilibrium either.

A.6. No Rat Race

This appendix repeats in our model the experiment of Section 2.3 of He and Xiong (2012*a*). In their model, a parameter changes and a creditor who takes the other creditors' old threshold as given raises its own threshold. Once every creditor has done the same, each creditor faces a higher threshold from the others and raises its own again, and the sequence of responses converges upward to the new equilibrium threshold. The first response covers only part of the distance, about one fourth in their illustration, and the mutual responses, the rat race, cover the rest. We show that in our model the individual response is at least as large as the equilibrium adjustment, so there is no rat race.

The experiment is an increase in the arrival rate of financing needs from λ_1 to $\lambda_2 > \lambda_1$, which raises the funding threshold by Proposition 2. We write $y_1 = y^*(\lambda_1)$ and $y_2 = y^*(\lambda_2)$ for the old and the new equilibrium thresholds, so $y_1 < y_2$. The argument below applies as it stands to any parameter change that raises the threshold.

Consider the active player at a round immediately after the increase in λ, who conjectures that every future round, inside or market, is funded if and only if the dividend is above the old threshold y_1. The conjecture covers both the owners at inside rounds and the new investors at market rounds.

Let $W(y; z, \lambda)$ be the equity value at dividend y when every future round is funded if and only if the dividend is at least z and financing needs arrive at rate λ. Repeating the derivation of Eq. (A.1) with the funding sets $F = F^o = [z, \infty)$ and combining the inside and market round

terms as in Eq. (6),

$$\rho W = \mu y\, W' + \frac{\sigma^2}{2} y^2\, W'' + \phi\,(y - W) + \lambda\,(1 - p\theta)\,(W - x(y))\,\mathbb{1}_{\{y \geq z\}} - \lambda W$$

where the argument $(y; z, \lambda)$ of W is omitted. The equilibrium equity value at the new rate is $W(\cdot; y_2, \lambda_2)$, since by Proposition 2 the indicator $\mathbb{1}_{\{y \geq y_2\}}$ equals $\mathbb{1}_{\{W \geq x\}}$ and the equation at $z = y_2$ is Eq. (6) at λ_2.

The active player funds as long as $W(y; y_1, \lambda_2) \geq x(y)$, so its response to the change is the funding set $\{y : W(y; y_1, \lambda_2) \geq x(y)\}$. We write $\hat{y}$ for the lowest dividend in this set and refer to $\hat{y}$ as the threshold under the stale conjecture.

Define $D(y) = W(y; y_2, \lambda_2) - W(y; y_1, \lambda_2)$. Subtracting the equation at $z = y_1$ from the equation at $z = y_2$, both at λ_2,

$$\rho D = \mu y\, D' + \frac{\sigma^2}{2} y^2\, D'' - (\phi + \lambda_2)\, D + \lambda_2\,(1 - p\theta)\Big[(W(y; y_2, \lambda_2) - x)^+ - (W(y; y_1, \lambda_2) - x)\,\mathbb{1}_{\{y \geq y_1\}}\Big]$$

Substituting $W(y; y_1, \lambda_2) = W(y; y_2, \lambda_2) - D(y)$ inside the bracket,

$$\rho D = \mu y\, D' + \frac{\sigma^2}{2} y^2\, D'' - (\phi + \lambda_2)\, D + \lambda_2\,(1 - p\theta)\, D\,\mathbb{1}_{\{y \geq y_1\}} + \lambda_2\,(1 - p\theta)\, h(y)$$

where

$$h(y) = (W(y; y_2, \lambda_2) - x(y))^+ - (W(y; y_2, \lambda_2) - x(y))\,\mathbb{1}_{\{y \geq y_1\}}$$

The function h is nonnegative. For $y < y_1$ both terms are zero. For $y \geq y_2$ the equity value is at least the cost of the round, so both terms equal $W(y; y_2, \lambda_2) - x(y)$ and h is zero. For $y_1 \leq y < y_2$ the equity value is below the cost of the round by Proposition 2, so the first term is zero and the second is negative, and $h(y) = x(y) - W(y; y_2, \lambda_2) > 0$. This is the region in which the stale conjecture funds rounds that create a negative surplus.

Collecting the terms in D on the left,

$$r(y)\, D - \mu y\, D' - \frac{\sigma^2}{2} y^2\, D'' = \lambda_2\,(1 - p\theta)\, h(y), \qquad r(y) = \rho + \phi + \lambda_2 - \lambda_2\,(1 - p\theta)\,\mathbb{1}_{\{y \geq y_1\}}$$

and $r(y) \geq \rho + \phi + \lambda_2 p\theta > 0$. Since D grows at most linearly and r is bounded below by a positive constant, the Feynman-Kac formula represents the solution as

$$D(y) = \mathbb{E}_y\left[\int_0^\infty e^{-\int_0^t r(y_s)\mathrm{d}s}\, \lambda_2\,(1 - p\theta)\, h(y_t)\,\mathrm{d}t\right] \geq 0$$

Hence $W(y; y_1, \lambda_2) \leq W(y; y_2, \lambda_2)$ for every y. Evaluating at $\hat{y}$, the lowest dividend at which the player funds under the stale conjecture, gives $W(\hat{y}; y_1, \lambda_2) \leq W(\hat{y}; y_2, \lambda_2)$. By the definition of

$\hat{y}$ the left side equals $x(\hat{y})$, so

$$x(\hat{y}) \leq W(\hat{y}; y_2, \lambda_2)$$

and $\hat{y} \geq y_2$, since by Proposition 2 the equilibrium equity value reaches the cost of the round only at dividends of at least y_2.

The individual response to the change, from y_1 to $\hat{y}$, is at least as large as the equilibrium adjustment from y_1 to y_2. In He and Xiong (2012*a*) the ordering is reversed. A creditor who expects the other creditors to run sooner runs sooner, the individual response falls short of the equilibrium adjustment, and the remaining gap is due to creditors reacting to others raising their thresholds. Here a player who expects the other players to fund less generously than in equilibrium funds more, so the responses dampen rather than amplify, and there is no rat race.

A.7. Derivation of the Total Surplus

We show that for any funding decisions $\left(d_j\right)_{j\geq 1}$ in $\{0,1\}$ and any stakes $(s_i)_{i\geq 1}$ in $[0,1]$, the sum of the discounted payoffs of the founder and of every new investor is

$$U_{0,t} + \sum_{i\geq 1} U_{i,t} = e^{-\rho\left(\tau_\phi - t\right)} y_{\tau_\phi} S\left(t, \tau_\phi\right) - \sum_{\substack{\text{all rounds } j \\ t<T_j\leq\tau_\phi}} e^{-\rho\left(T_j - t\right)} x\left(y_{T_j}\right) S\left(t, T_j\right) \tag{A.8}$$

path by path, which is the integrand of Eq. (7). We write the payoff of each player. The owners together put in $x\left(y_{T_j}\right)$ at a funded inside round, and the sum of their payoffs does not depend on how they split it. To fix ideas we take the split in an inside round to be in proportion to their holdings, as in Eq. (1).

The founder starts with share 1 of the firm and is diluted by $1 - s_j$ at every funded market round, so at any date u it holds share $D(t,u)$ of the firm. At every funded inside round it contributes $D\left(t, T_j\right) x\left(y_{T_j}\right)$, and if the firm survives to the payout it receives its share of the dividend. Discounting to t,

$$U_{0,t} = e^{-\rho\left(\tau_\phi - t\right)} y_{\tau_\phi} D\left(t, \tau_\phi\right) S\left(t, \tau_\phi\right) - \sum_{\substack{\text{inside } j \\ t<T_j\leq\tau_\phi}} e^{-\rho\left(T_j - t\right)} D\left(t, T_j\right) x\left(y_{T_j}\right) S\left(t, T_j\right) \tag{A.9}$$

Investor i is called at market round T_i and receives a payoff only if the firm has survived to T_i and that round is funded, which is the factor $S(t, T_i)$. It then contributes $x\left(y_{T_i}\right)$ and receives a stake s_i of the firm. From T_i on it is one of the owners with initial stake s_i. It is diluted by later market rounds, so it holds $s_i D(T_i, u)$ at date $u > T_i$, it contributes $s_i D\left(T_i, T_j\right) x\left(y_{T_j}\right)$ at every later funded inside round, and it receives its share of the dividend if the firm survives

to the payout. Hence

$$U_{i,t} = S(t,T_i)\Bigg[- e^{-\rho(T_i-t)} x\Big(y_{T_i}\Big) + s_i\, e^{-\rho\left(\tau_\phi - t\right)} y_{\tau_\phi}\, D\Big(T_i,\tau_\phi\Big)\, S\Big(T_i,\tau_\phi\Big) \\ - s_i \sum_{\substack{\text{inside } j \\ T_i<T_j\le\tau_\phi}} e^{-\rho\left(T_j-t\right)} D\Big(T_i,T_j\Big) x\Big(y_{T_j}\Big)\, S\Big(T_i,T_j\Big)\Bigg] \tag{A.10}$$

Investors whose round is an inside round never enter, and no investor is called after τ_ϕ, so the sum in Eq. (A.8) runs over the market rounds in $\Big(t,\tau_\phi\Big]$.

At any date u the firm's equity is held by the founder and by every investor whose market round was funded before u, with shares $D(t,u)$ and $s_i D(T_i,u)$. Therefore we have:

$$D(t,u) + \sum_{\substack{\text{market } i \\ t<T_i\le u}} s_i\, D(T_i,u) = 1 \tag{A.11}$$

Adding Eq. (A.9) and Eq. (A.10) over the market rounds and collecting the dividend terms, using $S(t,T_i)\,S\Big(T_i,\tau_\phi\Big) = S\Big(t,\tau_\phi\Big)$,

$$e^{-\rho\left(\tau_\phi - t\right)} y_{\tau_\phi}\, S\Big(t,\tau_\phi\Big)\Bigg[D\Big(t,\tau_\phi\Big) + \sum_{\substack{\text{market } i \\ t<T_i\le\tau_\phi}} s_i\, D\Big(T_i,\tau_\phi\Big)\Bigg] = e^{-\rho\left(\tau_\phi - t\right)} y_{\tau_\phi}\, S\Big(t,\tau_\phi\Big)$$

by Eq. (A.11) at $u = \tau_\phi$.

The remaining terms are

$$- \sum_{\substack{\text{inside } j \\ t<T_j\le\tau_\phi}} e^{-\rho\left(T_j-t\right)} D\Big(t,T_j\Big) x\Big(y_{T_j}\Big)\, S\Big(t,T_j\Big) \\ - \sum_{\substack{\text{market } i \\ t<T_i\le\tau_\phi}} \sum_{\substack{\text{inside } j \\ T_i<T_j\le\tau_\phi}} s_i\, e^{-\rho\left(T_j-t\right)} D\Big(T_i,T_j\Big) x\Big(y_{T_j}\Big)\, S\Big(t,T_j\Big) \\ - \sum_{\substack{\text{market } i \\ t<T_i\le\tau_\phi}} e^{-\rho(T_i-t)} x\Big(y_{T_i}\Big)\, S(t,T_i)$$

where in the double sum we used $S(t,T_i)\,S\Big(T_i,T_j\Big) = S\Big(t,T_j\Big)$. The double sum runs over the

pairs of a market round i and a later inside round j. Swapping the order of summation:

$$\sum_{\substack{\text{market } i \\ t<T_i\le\tau_\phi}} \sum_{\substack{\text{inside } j \\ T_i<T_j\le\tau_\phi}} s_i\, e^{-\rho\left(T_j-t\right)} D\left(T_i,T_j\right) x\left(y_{T_j}\right) S\left(t,T_j\right)$$
$$= \sum_{\substack{\text{inside } j \\ t<T_j\le\tau_\phi}} e^{-\rho\left(T_j-t\right)} x\left(y_{T_j}\right) S\left(t,T_j\right) \sum_{\substack{\text{market } i \\ t<T_i<T_j}} s_i\, D\left(T_i,T_j\right)$$

Grouping it with the first sum,

$$-\sum_{\substack{\text{inside } j \\ t<T_j\le\tau_\phi}} e^{-\rho\left(T_j-t\right)} x\left(y_{T_j}\right) S\left(t,T_j\right)\left[D\left(t,T_j\right)+\sum_{\substack{\text{market } i \\ t<T_i<T_j}} s_i\, D\left(T_i,T_j\right)\right] = -\sum_{\substack{\text{inside } j \\ t<T_j\le\tau_\phi}} e^{-\rho\left(T_j-t\right)} x\left(y_{T_j}\right) S\left(t,T_j\right)$$

by Eq. (A.11) at $u=T_j$. The full cost of every funded inside round appears once.

Collecting the dividend, the inside rounds and the market rounds,

$$U_{0,t}+\sum_{i\ge1} U_{i,t} = e^{-\rho\left(\tau_\phi-t\right)} y_{\tau_\phi}\, S\left(t,\tau_\phi\right) - \sum_{\substack{\text{inside } j \\ t<T_j\le\tau_\phi}} e^{-\rho\left(T_j-t\right)} x\left(y_{T_j}\right) S\left(t,T_j\right) - \sum_{\substack{\text{market } i \\ t<T_i\le\tau_\phi}} e^{-\rho(T_i-t)} x\left(y_{T_i}\right) S(t,T_i)$$

and the last two sums together run over every round in $\left(t,\tau_\phi\right]$, which is Eq. (A.8).

A.8. Proof of Proposition 4

Proof. Below u^* there is no entry, so Eq. (11) becomes

$$\frac{\sigma^2}{2} f''(u) - m f'(u) - (\phi+\lambda) f(u) = 0$$

Its solutions are $e^{\kappa u}$ with κ a root of $\frac{\sigma^2}{2}\kappa^2 - m\kappa - (\phi+\lambda) = 0$, whose roots have opposite signs. The negative root is ruled out, since $e^{\kappa u}$ would then diverge as $u\to-\infty$ and the mass of firms below any dividend would be infinite. Hence $f(u) = A\, e^{\kappa u}$ on $(-\infty,u^*)$ with κ the positive root in Eq. (12) and $A>0$ a constant pinned down by the solution above u^*. Failures occur at rate λ below u^*, so the total failure flow and the failure flow at dividends the benchmark would fund, those with $u\in[\ln y^*(0),\ln y^*(\theta))$, are

$$\lambda\int_{-\infty}^{\ln y^*(\theta)} A\, e^{\kappa u}\,\mathrm{d}u = \frac{A\lambda}{\kappa}\, y^*(\theta)^\kappa, \qquad \lambda\int_{\ln y^*(0)}^{\ln y^*(\theta)} A\, e^{\kappa u}\,\mathrm{d}u = \frac{A\lambda}{\kappa}\left[y^*(\theta)^\kappa - y^*(0)^\kappa\right]$$

Their ratio is Eq. (12). The constant A cancels, which removes η and g, and by Eq. (A.5) the slope α scales $y^*(0)$ and $y^*(\theta)$ by the same factor, so it cancels in the ratio as well. □

A.9. Proof of Proposition 5

Proof. Let $\ell(u)$ be the probability that a firm with log dividend u eventually fails, under a threshold u^*. Over a short interval $\mathrm{d}t$ one of three things happens. A financing need arrives, with probability $\lambda\,\mathrm{d}t$, and the firm fails if $u < u^*$ and otherwise is funded and continues from u. The payout arrives, with probability $\phi\,\mathrm{d}t$, and the firm never fails. Neither arrives, and the log dividend moves to $u + \mathrm{d}u$. Hence

$$\ell(u) = \lambda\,\mathrm{d}t\left[\mathbb{1}_{\{u<u^*\}} + \mathbb{1}_{\{u\geq u^*\}}\,\ell(u)\right] + (1-(\lambda+\phi)\,\mathrm{d}t)\,\mathbb{E}[\ell(u+\mathrm{d}u)]$$

A second-order expansion of $\ell(u+\mathrm{d}u)$ with $\mathrm{d}u = m\,\mathrm{d}t + \sigma\,\mathrm{d}Z_t$, using $\mathbb{E}[\mathrm{d}Z_t] = 0$ and $\mathbb{E}\left[\mathrm{d}Z_t^2\right] = \mathrm{d}t$ and dropping terms of order $\mathrm{d}t^2$, gives $\mathbb{E}[\ell(u+\mathrm{d}u)] = \ell(u) + m\,\ell'(u)\,\mathrm{d}t + \frac{\sigma^2}{2}\,\ell''(u)\,\mathrm{d}t$. Substituting, dividing by $\mathrm{d}t$ and taking the limit $\mathrm{d}t \to 0$ gives the following ordinary differential equation for ℓ:

$$0 = \frac{\sigma^2}{2}\,\ell'' + m\,\ell' - (\phi+\lambda)\,\ell + \lambda\left[\mathbb{1}_{\{u<u^*\}} + \mathbb{1}_{\{u\geq u^*\}}\,\ell\right] \tag{A.12}$$

Above u^*, Eq. (A.12) becomes $\frac{\sigma^2}{2}\ell'' + m\ell' - \phi\ell = 0$, whose solutions are e^{zu} with z a root of $\frac{\sigma^2}{2}z^2 + mz - \phi = 0$, one positive and one negative. A firm far above the threshold almost never fails, so $\ell(u) \to 0$ as $u \to \infty$ and the positive root is ruled out. Hence

$$\ell(u) = B\,e^{-\zeta u}, \qquad \zeta = \frac{m + \sqrt{m^2 + 2\sigma^2\phi}}{\sigma^2} > 0$$

where $-\zeta$ is the negative root. Below u^*, Eq. (A.12) becomes $\frac{\sigma^2}{2}\ell'' + m\ell' - (\phi+\lambda)\ell + \lambda = 0$. The constant $\lambda/(\lambda+\phi)$ is a particular solution, and it is the probability that the financing clock rings before the payout clock, which is the probability of failure for a firm so far below the threshold that it never returns. The homogeneous solutions are e^{zu} with z a root of $\frac{\sigma^2}{2}z^2 + mz - (\phi+\lambda) = 0$, and since ℓ is bounded as $u \to -\infty$, the negative root is ruled out. Hence

$$\ell(u) = \frac{\lambda}{\lambda+\phi} + A\,e^{r(u-u^*)}, \qquad r = \frac{-m + \sqrt{m^2 + 2\sigma^2(\phi+\lambda)}}{\sigma^2} > 0$$

Continuity of ℓ and ℓ' at u^*[24] gives $\frac{\lambda}{\lambda+\phi} + A = Be^{-\zeta u^*}$ and $rA = -\zeta Be^{-\zeta u^*}$. Eliminating A,

$$\ell(u) = \frac{\lambda}{\lambda+\phi}\,\frac{r}{r+\zeta}\,e^{-\zeta(u-u^*)}, \qquad u \geq u^* \tag{A.13}$$

To obtain the failure flow, consider a long horizon T. Over it, $\eta T g(u)\,\mathrm{d}u$ firms enter with log dividend in $[u, u+\mathrm{d}u)$, and each of them eventually fails with probability $\ell(u)$, so the number of firms that fail among those born over the horizon is $\eta T \int g(u)\ell(u)\,\mathrm{d}u$. In the stationary state the failure flow is this number per unit of time, the entry rate times the probability that an entrant eventually fails. Entrants have log dividends above $\ln y^*(\theta)$, where Eq. (A.13) applies, so with bargaining power the failure flow is

$$D_\theta = \eta \int_{\ln y^*(\theta)}^{\infty} g(u)\,\frac{\lambda}{\lambda+\phi}\,\frac{r}{r+\zeta}\,e^{-\zeta(u-\ln y^*(\theta))}\,\mathrm{d}u$$

Removing bargaining power lowers the threshold to $\ln y^*(0)$, which is below the support of g, so Eq. (A.13) still applies to every entrant and the failure flow is the same expression with $\ln y^*(0)$ in the exponent, D_0. The two integrands differ by the constant factor $e^{-\zeta[\ln y^*(\theta)-\ln y^*(0)]}$, so

$$\frac{D_\theta - D_0}{D_\theta} = 1 - \frac{D_0}{D_\theta} = 1 - \left(\frac{y^*(0)}{y^*(\theta)}\right)^{\zeta}$$

which is Eq. (13). □

A.10. Continuity of ℓ and ℓ' at the Threshold

We approximate the log dividend by a binomial process on a grid and show that the discrete analogues of ℓ and ℓ' are continuous at the threshold without imposing it. Fix a time step Δ, let $h = \sigma\sqrt{\Delta}$, and let the log dividend move from u_i to $u_i + h$ with probability ω and to $u_i - h$ with probability $1-\omega$, where

$$\omega = \frac{1}{2} + \frac{m}{2\sigma}\sqrt{\Delta}, \qquad m = \mu - \frac{\sigma^2}{2}$$

so that the step has mean $m\Delta$ and variance $\sigma^2\Delta$ up to terms of order Δ^2. Let i^* be the grid index of the threshold, $u_{i^*} = u^*$, and write ℓ_i for the probability that a firm with log dividend u_i eventually fails. Over one step the payout arrives with probability $\phi\Delta$, after which the firm never fails, a financing need arrives with probability $\lambda\Delta$, and otherwise the log dividend

[24] Appendix A.10 shows why ℓ and ℓ' are continuous at the threshold, by approximating the log dividend with a binomial process and taking the limit.

moves. Writing $a = (1-\lambda\Delta)(1-\phi\Delta)$ and $b = \lambda\Delta(1-\phi\Delta)$, the law of total probability gives

$$\ell_i = b\,\mathbb{1}_{\{i<i^*\}} + b\,\mathbb{1}_{\{i\geq i^*\}}\ell_i + a\left[\omega\ell_{i+1} + (1-\omega)\ell_{i-1}\right] \tag{A.14}$$

since below the threshold a financing need means failure and above it the round is funded and the firm continues from the same state.

For $i < i^*$, Eq. (A.14) is $\ell_i = b + a[\omega\ell_{i+1} + (1-\omega)\ell_{i-1}]$. A constant $\bar{\ell}$ solves it if $\bar{\ell}(1-a) = b$, that is,

$$\bar{\ell} = \frac{b}{1-a} = \frac{\lambda(1-\phi\Delta)}{\phi + \lambda - \lambda\phi\Delta}$$

which is the probability that the financing clock rings before the payout clock on the grid, the failure probability of a firm so far below the threshold that it never returns, and it converges to $\lambda/(\lambda+\phi)$ as $\Delta \to 0$. The homogeneous equation has solutions ξ^i with ξ a root of $a\omega\xi^2 - \xi + a(1-\omega) = 0$. The left side is positive at $\xi = 0$ and equals $a - 1 < 0$ at $\xi = 1$, so the roots satisfy $0 < \xi_-^U < 1 < \xi_+^U$. As $i \to -\infty$, $\left(\xi_-^U\right)^i$ diverges and ℓ_i would not be a probability, so

$$\ell_i^U = \bar{\ell} + B\left(\xi_+^U\right)^i$$

for some constant B.

For $i \geq i^*$, Eq. (A.14) is $(1-b)\ell_i = a[\omega\ell_{i+1} + (1-\omega)\ell_{i-1}]$, and its solutions are ξ^i with ξ a root of $a\omega\xi^2 - (1-b)\xi + a(1-\omega) = 0$. The left side is positive at $\xi = 0$ and equals $a - (1-b) = -\phi\Delta < 0$ at $\xi = 1$, so again $0 < \xi_-^F < 1 < \xi_+^F$. As $i \to \infty$ the failure probability vanishes, which rules out ξ_+^F, so

$$\ell_i^F = A\left(\xi_-^F\right)^i$$

for some constant A.

The two branches meet at i^*, and the equations at the two nodes adjacent to the threshold each reach across it. At $i = i^*$ the funded equation is $(1-b)\ell_{i^*}^F = a\omega\,\ell_{i^*+1}^F + a(1-\omega)\ell_{i^*-1}^U$, and solving for the unfunded value,

$$\ell_{i^*-1}^U = \frac{(1-b)\ell_{i^*}^F - a\omega\,\ell_{i^*+1}^F}{a(1-\omega)} = A\left(\xi_-^F\right)^{i^*-1}\frac{(1-b)\xi_-^F - a\omega\left(\xi_-^F\right)^2}{a(1-\omega)} = A\left(\xi_-^F\right)^{i^*-1} = \ell_{i^*-1}^F$$

where the third equality uses the characteristic equation of ξ_-^F. At $i = i^* - 1$ the unfunded equation is $\ell_{i^*-1}^U = b + a\omega\,\ell_{i^*}^F + a(1-\omega)\ell_{i^*-2}^U$, and solving for the funded value,

$$\ell_{i^*}^F = \frac{\ell_{i^*-1}^U - a(1-\omega)\ell_{i^*-2}^U - b}{a\omega} = \frac{\bar{\ell}[1-a(1-\omega)] - b}{a\omega} + B\left(\xi_+^U\right)^{i^*-2}\frac{\xi_+^U - a(1-\omega)}{a\omega} = \bar{\ell} + B\left(\xi_+^U\right)^{i^*} = \ell_{i^*}^U$$

The two branches therefore agree at the two adjacent nodes i^*-1 and i^*, so their values and

their difference quotients $(\ell_{i^*} - \ell_{i^*-1})/h$ coincide. Neither condition is imposed. Both follow from the recursion at the nodes next to the threshold. As $\Delta \to 0$ the two branches converge to the two solutions of Eq. (A.12) on either side of u^*, and the two seam conditions become the continuity of ℓ and of ℓ' at u^* used in the proof of Proposition 5. The same argument, applied to the equity value on the grid, gives the continuity of W and W' at y^* used in the proof of Proposition 2.

B Data

Table 5 reports summary statistics for the firms in the estimation sample of Section 4.5 and for their priced rounds.

C Estimation Details

This appendix describes the simulation behind the estimator of Section 5.3. Given a candidate (μ, σ) and the parameters already in hand, the threshold and the equity value are available in closed form from Appendix A.4, so nothing is solved numerically except the inversion of the entry states below. The simulation then produces an event-level panel with the same columns as the data, and the two slopes of Eq. (17) are computed on the simulated panel by the same code that computes them on the data.

C.1. Entry States

The dividend at entry, y_0, is never observed, but every firm in the estimation sample has an observed stake sold at its first round, s_1, and the first round is a market round. By Eq. (3), the stake sold at a market round is a strictly decreasing function of the dividend, from one at the threshold toward θ as the dividend grows, since the firm's value rises relative to the constant cost of the round. A firm's entry state is therefore the dividend that reproduces its entry stake, the solution y_0 of

$$s_1 = \theta + (1 - \theta)\,\frac{x_0}{W(y_0)} \tag{C.1}$$

which we find by solving the equation above numerically. Since y^* and W depend on (μ, σ), the inversion is repeated at every evaluation of the objective, once for each of the 16,521 firms in the sample. A stake at or below θ cannot be attained at any finite dividend. For the firms whose entry stake falls there, about one percent of the sample, the stake is set just above θ and the entry state is capped at the upper end of the bisection interval, so that the firm is simulated as born far above the threshold.

Table 5: Summary statistics of the estimation sample

	Mean	SD	p10	Median	p90	N
Panel A: Firms (16,521)						
Sector (% of firms)						
Information technology	46.3					16,521
Healthcare	21.4					16,521
Consumer products and services	15.6					16,521
Business products and services	11.7					16,521
Other	5.1					16,521
Headquarters (% of firms)						
California	41.1					16,521
New York	11.1					16,521
Massachusetts	8.6					16,521
Terminal outcome by October 2025 (% of firms)						
IPO	4.1					16,521
Acquired	35.0					16,521
Failed	29.7					16,521
Still active (censored)	31.2					16,521
Firm characteristics						
Year founded	2011	5	2005	2012	2016	16,223
Year of first round	2013	4	2007	2014	2018	16,521
Age at first round (years)	2.4	4.0	0.0	1.0	5.0	16,219
First-round raise ($m)	4.91	15.48	0.40	2.15	10.00	16,465
First-round post-money ($m)	18.0	140.7	2.3	8.7	31.6	16,355
First-round stake sold (%)	29.0	15.7	10.4	26.8	49.3	16,521
Priced rounds observed	3.1	2.0	1.0	3.0	6.0	16,521
Total equity raised ($m)	54.7	265.4	1.0	11.3	118.1	16,482
Years observed	12.1	3.7	7.8	11.4	18.0	16,521
Panel B: Priced financing rounds (51,594)						
Stage (% of rounds)						
Seed	18.2					51,593
Early stage	38.6					51,593
Late stage	43.2					51,593
Lead investor recorded	37.8					51,594
Led by a new investor (market round)	82.4					19,521
Led by a new investor, rounds 2 and later	76.4					14,227
Down round	14.1					25,569
Round characteristics						
Round number	2.7	1.8	1.0	2.0	5.0	51,594
Number of investors	4.7	4.6	1.0	3.0	10.0	42,411
Years since previous round	1.81	1.34	0.56	1.49	3.38	35,073
Firm age at round (years)	4.9	4.4	1.0	4.0	10.0	50,749
Financing year	2016	5	2009	2016	2021	51,594
Equity raised ($m)	18.3	104.4	0.6	5.0	38.1	49,268
Pre-money valuation ($m)	139.8	1,284.5	3.0	15.6	175.0	43,457
Post-money valuation ($m)	160.6	1,365.3	4.2	22.1	213.7	43,537
Stake sold (%)	24.5	14.7	7.7	22.4	43.9	43,756

C.2. Draws

Each firm in the sample is simulated $S = 10$ times, following Michaelides and Ng (2000). A simulated firm is paired with a data firm drawn uniformly with replacement and inherits that firm's entry stake s_1, hence its entry state, and its observation window $\bar{T}$, the years from its first priced round to the end of the data. The pairing reproduces the right-censoring structure of the data in the simulated panel. All random draws are made once, before the estimation starts, and are kept fixed across candidate parameters, so that the objective is a deterministic function of (μ, σ) and the optimizer changes parameters but not the draws. Table 6 lists the draws behind one simulated firm and what each becomes. The buffer K is chosen so that no simulated firm can exhaust its financing gaps within the longest observation window in the data.

Table 6: The draws behind one simulated firm

Draw	Size	Becomes	Via
$u^{exit} \sim U(0,1)$	scalar	payout time	$\tau_\phi = -\ln u^{exit}/\phi$
$u_k^{gap} \sim U(0,1)$	K-vector	gap to financing need k	$\Delta t_k = -\ln u_k^{gap}/\lambda$
$u_k^{type} \sim U(0,1)$	K-vector	type of round k	market round if $u_k^{type} < p$
$z_k \sim N(0,1)$	$(K+1)$-vector	dividend step to event k	$y_{k+1} = y_k\, e^{(\mu-\sigma^2/2)\Delta t_k + \sigma\sqrt{\Delta t_k}\, z_k}$

C.3. Simulation of a Firm and the Panel

A firm is simulated in event time, with one exact step of the geometric Brownian motion between consecutive events rather than a time grid. The firm enters at $t = 0$ with dividend y_0, and its entry round is recorded with the model's pricing at y_0, so its stake is s_1 by construction. Its payout time is drawn from u^{exit}. Then, from the current date and dividend, the next financing need arrives after the next gap, and the next event is whichever comes first of three.

(i) Censoring, when the observation window $\bar{T}$ ends first. The firm's last row is censored and its dividend is not updated.

(ii) The payout, when τ_ϕ comes first. The dividend is advanced over the elapsed time with the GBM draw, and the firm's last row records the payout with the dividend at that date as its exit value.

(iii) A financing need, when the gap comes first. The dividend is advanced over the gap. If it is at or above the threshold, the round is funded. The type of the round comes from the uniform draw between 0 and 1. If it is a market round, it is priced by Eq. (3), and

otherwise it is an inside round priced at the equity value. The row records the cost of the round, the stake, the post-money valuation and the type of the round, and the loop continues. If the dividend is below the threshold, the firm fails and its last row records the failure.

A simulated firm fails only at a financing need that arrives while its dividend is below the threshold, as in the model. Stacking the rows of every simulated firm gives an event-level panel with the same columns as the data panel of Section 4, the entry round, the later rounds with their stakes and valuations, and a terminal row for the payout, the failure or the censoring. The regressions of Eq. (17) are then run on the simulated firms that reach the payout, exactly as they are run on the data firms that exit with a valuation, so the two slopes are subject to the same selection on both sides.

C.4. Alternative Definitions of a Market Round

Table 7 varies the definition of a market round in Eq. (15), with the controls and the fixed effects of the baseline. Column (1) is the baseline, in which a round is a market round when its lead investor is new to the firm. Column (2) replaces the indicator with the fraction of the round's investors that are new to the firm. If the bargaining power of the syndicate is θ times that fraction, the coefficient on the fraction is approximately $-\theta$, so bargaining power is recovered as minus the coefficient. Column (3) uses an indicator that at least one investor in the round is new to the firm, whether or not that investor leads. Columns (2) and (3) are estimated on the rounds with a recorded list of participating investors, about twice as many as the rounds with a recorded lead.

Table 7: Bargaining power under alternative definitions of a market round

	(1)	(2)	(3)
New lead investor	-0.0186^{***}		
	(0.0044)		
Fraction of new investors		-0.0244^{***}	
		(0.0053)	
Any new investor			-0.0154^{***}
			(0.0055)
log amount raised	-0.1013^{***}	-0.1108^{***}	-0.1098^{***}
	(0.0042)	(0.0025)	(0.0025)
log previous post-money	0.0666^{***}	0.0710^{***}	0.0711^{***}
	(0.0042)	(0.0030)	(0.0030)
Headcount growth	0.0306^{***}	0.0316^{***}	0.0314^{***}
	(0.0052)	(0.0036)	(0.0036)
Implied θ (%)	1.85	2.44	1.53
	(0.44)	(0.53)	(0.54)
Mapping	$1 - e^{\beta}$	$-\beta$	$1 - e^{\beta}$
Round-number FE	✓	✓	✓
Year × Industry FE	✓	✓	✓
Company FE	✓	✓	✓
Sample	Recorded lead	Recorded participants	Recorded participants
Observations	7,875	16,145	16,145
R^2	0.724	0.664	0.664

OLS estimates of $\log(1 - s_{ik}) = \beta D_{ik} + \Gamma' X_{ik} + \gamma_{r(ik)} + \delta_{t(ik)\times c(i)} + \chi_i + \varepsilon_{ik}$ on priced rounds $k \geq 2$ of company i with an observed stake s_{ik} and a reported raise x_{ik}; fixed effects as in the previous table. The controls X_{ik} are the baseline ones: the log amount raised, the log post-money valuation of the company's previous priced round, and the log change in its headcount between that round and this one (Revelio Labs), on the rounds with both. D_{ik} is an indicator that the round's lead investor is new to the company in column (1), the baseline; the fraction of the round's investors that are new to the company in column (2); and an indicator that at least one investor is new in column (3). Column (1) uses rounds with a recorded lead investor, columns (2) and (3) rounds with a recorded list of participating investors. Implied θ in percent, with delta-method standard errors: $\theta = 1 - e^{\beta}$ for an indicator, and $\theta = -\beta$ for the fraction, when the syndicate's bargaining weight is θf, since then $\log(1 - s) = \log(1 - x/W) + \log(1 - \theta f) \approx \log(1 - x/W) - \theta f$. Standard errors clustered by company in parentheses; $^{*}p < 0.10$, $^{**}p < 0.05$, $^{***}p < 0.01$.

C.5. Robustness of the Estimate of Bargaining Power

Table 8 reports the two robustness checks discussed in Section 5.1. Column (1) repeats the baseline, column (5) of Table 2. Column (2) adds a fixed effect for the lead investor, on the rounds whose lead investor leads at least two rounds in the sample. In a round with several leads, the lead is a new one at a market round and a returning one at an inside round, the most active of them in the sample when there is more than one. Column (3) adds indicators for a liquidation preference above one times the investment, for participating preferred stock and for full-ratchet anti-dilution, on the rounds for which these terms are observed. The second row of estimates applies the baseline specification to the rounds of each column, so that, within a column, the difference between the two rows comes only from the added controls.

Table 8: Robustness to the lead investor and to the terms of the round

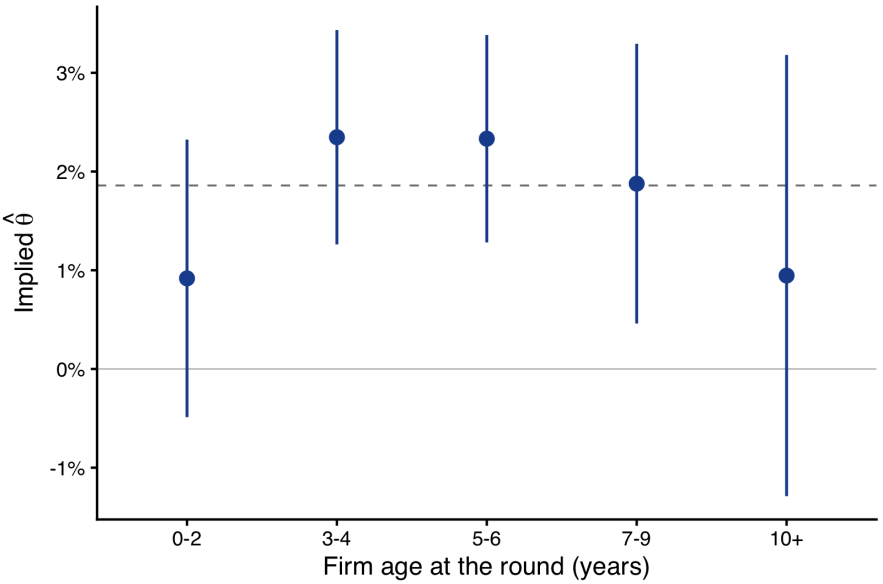
	(1)	(2)	(3)
Market round	−0.0186***	−0.0190***	−0.0174***
	(0.0044)	(0.0053)	(0.0043)
log amount raised	−0.1013***	−0.0895***	−0.1017***
	(0.0042)	(0.0053)	(0.0042)
log previous post-money	0.0666***	0.0592***	0.0643***
	(0.0042)	(0.0046)	(0.0044)
Headcount growth	0.0306***	0.0174***	0.0216***
	(0.0052)	(0.0056)	(0.0048)
Liquidation preference above 1x			0.0002
			(0.0271)
Participating preferred			−0.0112
			(0.0191)
Full-ratchet anti-dilution			−0.0223
			(0.0705)
Implied θ (%)	1.85	1.89	1.73
	(0.44)	(0.52)	(0.42)
baseline specification, same rounds		2.14	1.73
		(0.49)	(0.42)
Round-number FE	✓	✓	✓
Year × Industry FE	✓	✓	✓
Company FE	✓	✓	✓
Lead investor FE		✓	
Sample	Revelio matched	Repeat leads	Terms observed
Observations	7,875	5,911	6,931
R^2	0.724	0.831	0.751

OLS estimates of $\log(1 - s_{ik}) = \beta M_{ik} + \Gamma' X_{ik} + \gamma_{r(ik)} + \delta_{t(ik)\times c(i)} + \chi_i + \varepsilon_{ik}$ with additional controls, on priced rounds $k \geq 2$ of company i with a recorded lead investor, an observed stake s_{ik} and a reported raise x_{ik}; fixed effects as in the previous table. Column (1) is the baseline, column (5) of the previous table: the controls X_{ik} are the log amount raised, the log post-money valuation of the company's previous priced round and the log change in its headcount between that round and this one (Revelio Labs). Column (2) adds lead investor fixed effects, on the rounds whose lead investor leads at least two rounds; in rounds with several leads the lead is a new one in market rounds and a returning one in inside rounds (the most active in the sample if several). Column (3) adds indicators for a liquidation preference above 1x, participating preferred stock and full-ratchet anti-dilution, on the rounds with terms observed. Implied $\theta = 1 - e^{\beta}$ in percent, with delta-method standard errors. The second θ row re-estimates the baseline specification on the rounds of each column: within a column the difference between the two rows is due to the added terms, and along the second row the difference from column (1) is due to the sample. On the 5,265 rounds common to all columns, the baseline specification gives $\theta = 1.92$ (0.46). Standard errors clustered by company in parentheses; * $p < 0.10$, ** $p < 0.05$, *** $p < 0.01$.

C.6. Bargaining Power by Firm Age

Figure 5 reports the estimates of θ_A from the regression in Section 5.1, on the baseline rounds with an observed founding year.

Figure 5: Bargaining power by firm age



Note: Implied $\theta_A = 1 - e^{\beta_A}$ by the age of the firm at the round, with 95% confidence intervals. The dashed line is the baseline estimate.

D Sensitivity of the Cost of Bargaining Power

Figure 6 traces the two shares of Propositions 4 and 5 as new investors' bargaining power varies from zero to 5%, holding every other parameter at its estimated value.

Figure 6: The cost of bargaining power as a function of θ

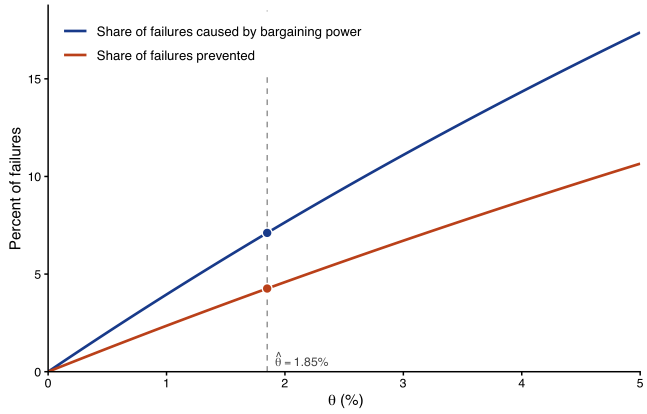
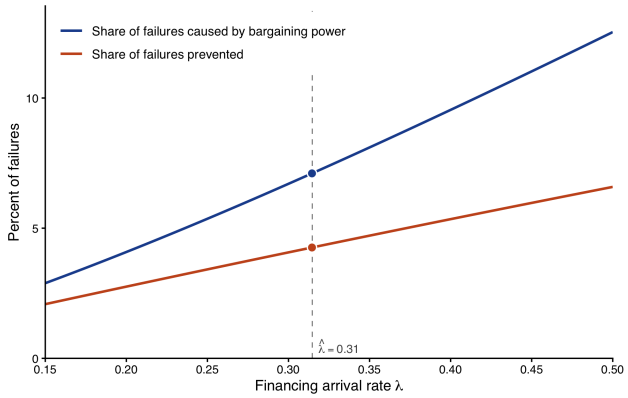


Figure 7 does the same for the other parameters, one at a time. Both shares rise with the

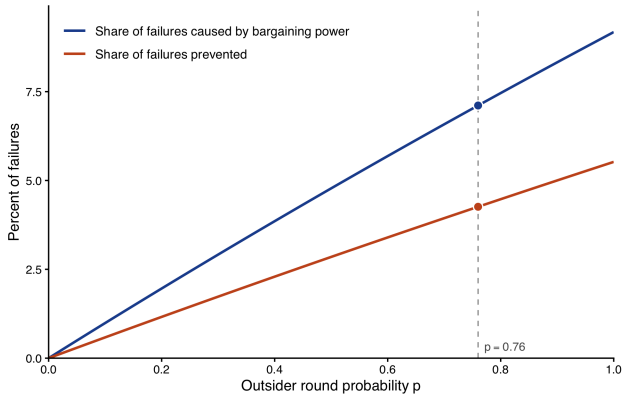
arrival rate of financing needs and with the probability of a market round, which add future market rounds at which new investors capture part of the surplus, and fall with the arrival rate of the payout and with the discount rate, which make those rounds matter less for the equity value today.

Figure 7: The cost of bargaining power as a function of the other parameters

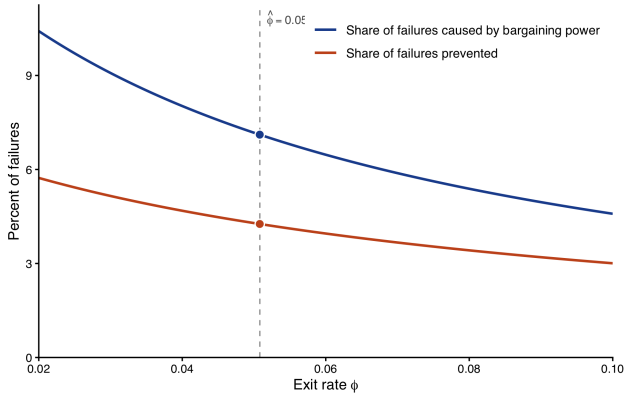
(a) Arrival rate of financing needs λ



(b) Probability of a market round p



(c) Arrival rate of the payout ϕ



(d) Discount rate ρ

