An introduction to Causality and Growth Rates

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Today's plan:

- An introduction to causality

- Dealing with Growth Rates

- Review of Slopes

Causality

Economists are often interested in answering the following question: Does an event A cause an event B?

Some examples:

- Does smoking (Event A) increase the chance of developing lung cancer (Event B)?
- Does democracy (Event A) cause growth (Event B)?

In most cases, it is really hard to answer these questions. Seeing that both events happen (correlation) is not sufficient! Why?

- Omitted variable
- Reverse causality

Causality

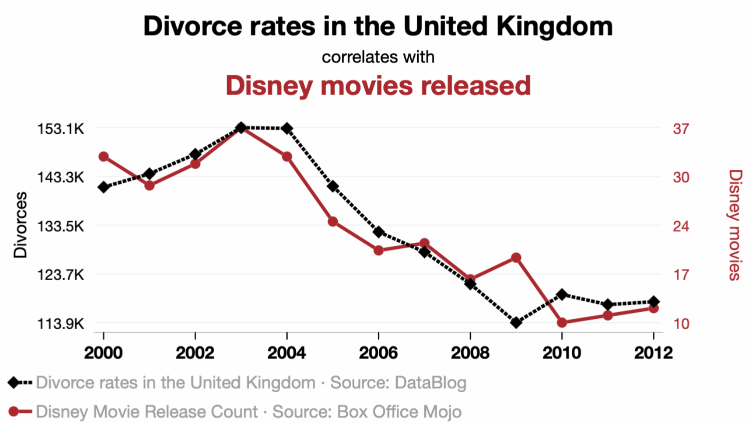
Omitted variable

- A study group observes a strong relationship between two variables: the number of cigarette lighters that a person owns and the probability that they will develop lung cancer in the future.
- The study group advises the government to discourage people from owning cigarette lighters.
- Problem: There is a third variable that is not being considered: smoking!
  - People who smoke are more likely to own cigarette lighters and also more likely to develop lung cancer.

Reverse causality:

- One study plots the number of violent crimes per thousand people in major cities against the number of police officers per thousand people.
- The study notes the curve being upward sloping and argues that because police increase rather than decrease the amount of urban violence, law enforcement should be abolished.
- Problem: More dangerous cities have more police officers!
  - So rather than police causing crime, crime causes police.

Correlation does not imply causation!!!



Source: https://www.tylervigen.com/spurious-correlations

Correlation does not imply causation!!!

Funny video

Growth rates

If some variable Y grows at a constant rate g, then there is an easy way to find the value of Y at some instant t_1 given its value at some instant t_0:

$$\boxed{Y_{t_1} = Y_{t_0} \cdot (1+g)^{t_1 - t_0}}$$

Let's see some examples:

- Suppose the US GDP grows at a constant rate of 2%. How long will it take US to double its GDP?

- US GDP grows at a constant rate of 2% while China's GDP grows at a constant rate of 5%. China's GDP today is 12.6% of US GDP. How long it will take China to surpass the US?

A useful approximation

We can generalize the result we found before.

Suppose US GDP grows at a constant rate of g and you want to find how long it will take for it to be x times the value it is today. Then:

$$t = \frac{\ln(x)}{\ln(1+g)}$$

Now, for small values of g, we can use the following beautiful approximation:

$$\ln(1+g) \approx g$$

Which gives us:

$$\boxed{t = \frac{\ln(x)}{g}}$$

⚠ Make sure you are using natural logarithm when applying this approximation. Otherwise you are going to be very sad with the result.

Slopes

The slope of a line is the "Rise" over the "Run".

$$\text{slope} = \frac{\text{Rise}}{\text{Run}} = \frac{\Delta Y}{\Delta X} = \frac{Y_2 - Y_1}{X_2 - X_1}$$

Important: We will use the symbol Δ a lot in this class!

- It means change in a variable between two scenarios (or two points in time).
- For example, ΔY means change in Y, which is equal to $Y_2 - Y_1$.

Consider a line represented by the equation $Y = a + b \cdot X$, then the slope of this line is b.

Why should I care about this?

Slopes

Suppose a variable Y grows at a constant rate of 10% each year and at $t = 0$ $Y_0 = 100$. Using the formula we just learned, Y_t is given by:

$$Y_t = 100 \cdot 1.1^t$$

Taking ln() on both sides:

$$\ln(Y_t) = \ln(100) + t \ln(1.1)$$

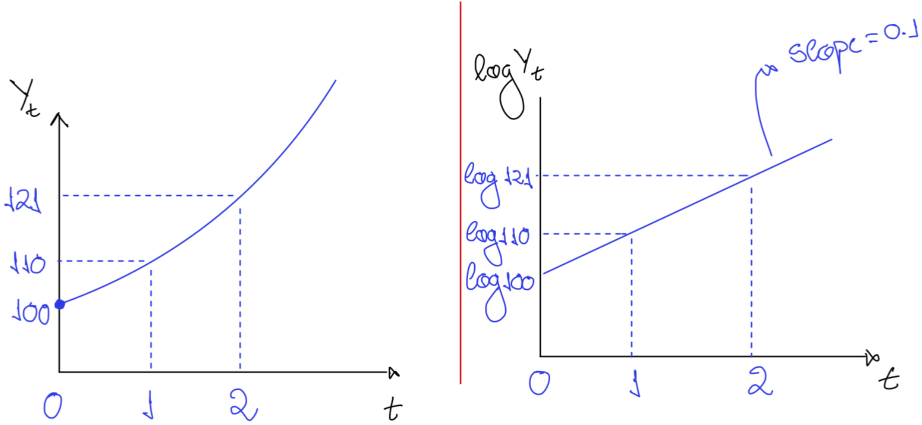
which is a line with slope ln(1.1).

Using the beautiful approximation we just learned: $\ln(1 + 0.1) \approx 0.1$. Then:

$$\ln(Y_t) = \ln(100) + 0.1 \cdot t$$

The slope of the line in the log-scale plot is approximately the growth rate!

Slopes



Average Growth Rates

Suppose we have a time series on a variable x, represented by: x_1, x_2, x_3, where the subscript refers to time.

The rate of growth of x between $t = 1$ and $t = 2$, g_1, is:

$$1 + g_1 = \frac{x_2}{x_1}$$

and the rate of growth of x between $t = 2$ and $t = 3$, g_2, is:

$$1 + g_2 = \frac{x_3}{x_2}$$

Note that:

$$x_1 \cdot (1 + g_1) \cdot (1 + g_2) = x_1 \cdot \frac{x_2}{x_1} \cdot \frac{x_3}{x_2} = x_3$$

Average Growth Rates

This gives us a motivation to find the geometric average rate of growth of x. Let's call it $\bar{g}$.
$\bar{g}$ is a constant growth rate that satisfies:

$$x_1 \cdot \underbrace{(1+\bar{g}) \cdot (1+\bar{g})}_{(1+\bar{g})^2} = x_3$$

Isolating $\bar{g}$:

$$\bar{g} = \left(\frac{x_3}{x_1}\right)^{1/2} - 1$$

What is the interpretation of $\bar{g}$?

- If we start with x_1 and grow $\bar{g}$ each period, we would end up with x_3 after two periods.

We can also express $\bar{g}$ in terms of the rate of growth of x for each period.

$$\cancel{x_1} \cdot (1+\bar{g})^2 = x_3 = \cancel{x_1} \cdot (1+g_1) \cdot (1+g_2) \Rightarrow$$
$$(1+\bar{g})^2 = (1+g_1) \cdot (1+g_2) \Rightarrow$$
$$\bar{g} = [(1+g_1) \cdot (1+g_2)]^{1/2} - 1$$

Average Growth Rates

Let's now generalize this result.

Suppose we have a time series on the variable x, represented by: $x_1, x_2, x_3 \ldots x_N$, where the subscript refers to time.

The rate of growth of x between $t = i$ and $t = i + 1$, g_i, is:

$$1 + g_i = \frac{x_{i+1}}{x_i}$$

The geometric average rate of growth of x, $\bar{g}$, is:

$$\bar{g} = \left(\frac{x_N}{x_1}\right)^{\frac{1}{N-1}} - 1$$

and expressed in terms of the rate of growth of x for each period:

$$\bar{g} = \left[\prod_{i=1}^{N-1}(1 + g_i)\right]^{\frac{1}{N-1}} - 1$$