

Closed Economy Model

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Today's plan:

- Why do we need to study models?
- Exogenous versus Endogenous variables
- Closed Economy Model.

Introduction

In the previous lectures, we focused on macroeconomic data.

Today, we will begin to introduce **macroeconomic theory**!

Our goal is to explain the data we see.

Can't we just look at the data?

- As social scientists, it is our job to try to understand **why** the macroeconomic variables move over time the way they do.
- We also need to think about **how policy would change** the macroeconomic variables.
 - This we what we call **counterfactuals**: what would have been true under different **circumstances**.

Before we jump into the closed economy model, let me define a concept called **Exogeneity**.

Exogenous versus Endogenous variables

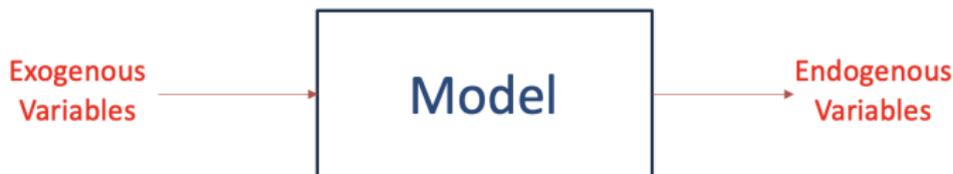
Any variable (Y, C, G, I...) in a model can be characterized as **Endogenous** or **Exogenous**.

Exogenous variables:

- Someone else (me in this class) tells you their values.
- Our model **does not predict** their values.
- They are the **input** of the model.

Endogenous variables:

- Those we are interested in.
- Given our exogenous variables, **our model predicts their values**.



Later, we will classify the variables in our closed economy model using this definition!

Closed Economy Model - Big Picture

We will focus on how **production is determined** and how the **income from production is distributed**.

We will discuss:

- **Production function**: technology for how to go from **inputs** of production to **output**.
- How **firms behave** when they maximize profits.
- How the different **inputs** are **rewarded** in the market;
- How **income** is **distributed**.
- How **output is used**.
- **Equilibrium** in the **market for goods** and in the **market for loans**.



We will focus on the **long-run**!

- We ignore, for the moment, unemployment and reasons why not all our resources would be fully used.
- This is referred to as **neoclassical theory**.

Aggregate Production Function

We will assume there is only **one good** in the economy, representing the total production.

- I will also refer to production as **GDP** and **output**.

Production takes place using capital **K** and labor **L**

- **Capital:**
 - The set of **tools** workers use: computers, calculators, machines, trucks ...
 - It's the result of **past decisions** (past investment).
- **Labor:** The **time people spend working** or the **number of workers**.

Production Function:

- Determines how much **output** Y is produced from given amounts of capital K and labor L .
- This relationship between output Y and inputs K and L is described by a function F .

$$Y = F(K, L)$$

Production Function - Assumptions

We will make the following **assumptions** about the production function.

- Constant Returns to Scale
- Positive Marginal Products
- Decreasing Marginal Products

1) Constant returns to scale:

- If we **multiply both** the amount of capital and the amount of labor by some number z , **output** is also **multiplied by z** .
- **Example:** **Doubling** the amount of **equipment** (K) and the number of **workers** (L) **doubles** the amount of **output** (Y).
- Formally, we can write this as:

$$\forall z \geq 0, \quad F(zK, zL) = zF(K, L)$$

Production Function - Assumptions

The **Marginal Product of Labor (MPL)** is defined as:

$$\text{MPL} = \frac{\partial F(K, L)}{\partial L}$$

- It is the **slope** of the production function when we **hold K fixed!**
- It tells you how much **output** will **increase**, approximately, if we increase the amount of **labor by one**.

The **Marginal Product of Capital (MPK)** is defined as:

$$\text{MPK} = \frac{\partial F(K, L)}{\partial K}$$

- It is the slope of the production function when we **hold L fixed!**
- It tells you how much **output** will increase, approximately, if we increase the amount of **capital one**.

Production Function - Assumptions

2) **Positive Marginal Products**: We will assume that:

$$MPK = \frac{\partial F(K, L)}{\partial K} > 0$$

$$MPL = \frac{\partial F(K, L)}{\partial L} > 0$$

What does this mean?

- Adding **additional workers** or **additional capital** to a productive process **increases** at least a little bit **total output**.

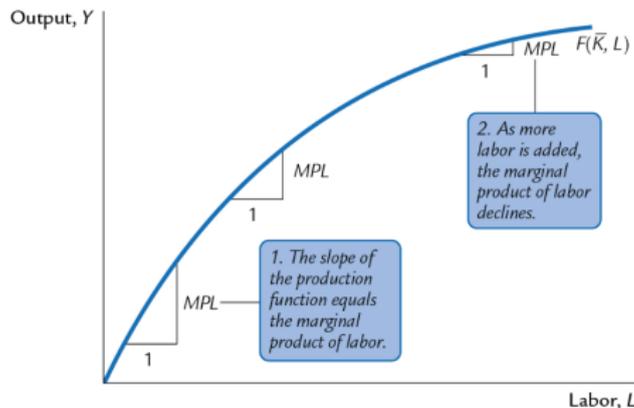
3) **Decreasing Marginal Products**:

- We will assume that **MPL is decreasing in L**.
- We will assume that **MPK is decreasing in K**.

Production Function - Assumptions

MPL is decreasing in L:

- Think about the production of bread in a bakery.
- As a bakery hires more labor, it produces more bread.
- As more labor is added to a fixed amount of capital, however, **fewer extra** units of bread are produced since the kitchen is getting **crowded!**
- These fewer extra units of bread produced when we increase L are captured by **MPL** being a **decreasing function of L**.



Production Function - Assumptions

MPK is decreasing in K:

- Again, consider the production of bread at a bakery.
- The first ovens installed in the kitchen will be very productive.
- But, if the bakery **installs more and more ovens, while holding L constant**, it will eventually **contain more ovens than its employees** can operate.
- Hence, the **marginal product** of the **last few** ovens is **lower** than that of the **first few**.

These assumptions regarding decreasing marginal products can be expressed as:

$$\frac{\partial \text{MPL}}{\partial L} = \frac{\partial^2 F(K, L)}{\partial L^2} < 0$$
$$\frac{\partial \text{MPK}}{\partial K} = \frac{\partial^2 F(K, L)}{\partial K^2} < 0$$

Output and input markets

Firm produces output in a **market for output**:

- Firm **sells** its production in this market.
- Households **buy** their production in this market.
- **Price of output**: P .

Firm buys/rents inputs in a **factor market**:

- **Labor's price**: W . The **wage** paid to households working at this firm.
- **Capital's price**: R . The **rental rate** charged for the machines this firm uses.
- In reality, most firms buy and own their capital, but here, we think of firms renting their capital.

We will assume that both markets are **perfectly competitive**!

- Firm cannot influence any price!
- The sellers (households) of factor inputs cannot influence any price.
- In other words, both **take the prices** in this **economy as given**!

Firm behavior

The firm is assumed to **maximize its profits**:

$$\begin{aligned}\text{Profit} &= \text{Revenues} - \text{Costs} \\ &= P \cdot F(K, L) - W \cdot L - R \cdot K\end{aligned}$$

- $P \cdot F(K, L)$: Firm's revenue from selling the output produced.
- $W \cdot L$: Labor costs.
- $R \cdot K$: Cost of renting capital.

Under perfect competition, the firm **takes** P , W , and R as given and **chooses** L and K that **maximize profits**.

Firm behavior

The firm's profit maximization problem can be written as:

$$\max_{K,L} PF(K, L) - WL - RK$$

Finding the optimal K and L is easy: the beautiful **First Order Conditions** (FOC):

- Take the **derivative** of the **firm's profit** with respect to L and **set it equal to 0**
- Take the **derivative** of the **firm's profit** with respect to K and **set it equal to 0**

First-order condition for **labor choice**:

$$P \frac{\partial F(K, L)}{\partial L} - W = 0 \Rightarrow$$

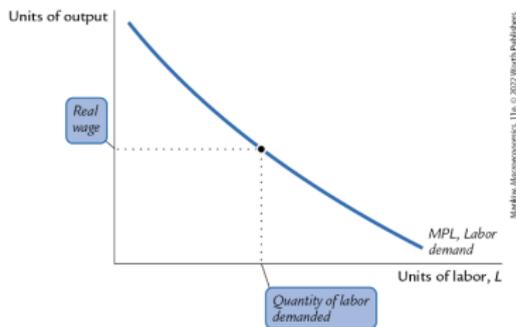
$$\text{MPL} = \frac{\partial F(K, L)}{\partial L} = \frac{W}{P}$$

(1)

Firm behavior

This **optimality condition** tells you that the firm hires labor until:

- The **marginal benefit** of hiring an additional unit of labor (MPL) is equal to the **marginal cost** of hiring an additional unit of labor, in real terms, (W/P).
 - W/P : is the **real wage**: It tells you how much it costs, in terms of **units of the output**, to hire a unit of labor.
- Note that in Eq. (1), for a given W/P , MPL gives you how much labor the firm will demand.
- That's why we also refer to the MPL as the **firm's labor demand curve**.



Firm Behavior

First-order condition for **capital choice**:

$$P \frac{\partial F(K, L)}{\partial K} - R = 0 \Rightarrow$$

$$\boxed{\text{MPK} = \frac{\partial F(K, L)}{\partial K} = \frac{R}{P}} \quad (2)$$

This **optimality condition** tells you that the firm rents K until:

- The **marginal benefit** of renting an additional unit of K (**MPK**) is equal to the **marginal cost** of renting an additional unit of K , in real terms, (R/P).
- R/P : is the **real rental price of capital**: It tells you how much it costs, in terms of **units of the output**, to rent a unit of capital.

Firm Behavior

- Note that in Eq. (2), for a given R/P , MPK gives you how much labor the firm will demand.
- That's why we also refer to the MPK as the firm's capital demand curve.

Key Takeaway:

The firm demands each factor of production until that factor's marginal product equals its real factor price.

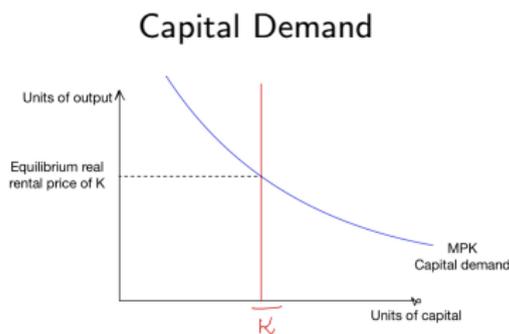
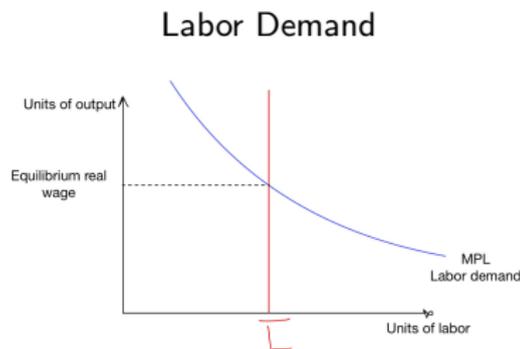
Factor's Price

We will assume that the supply of factors is fixed at \bar{L} and \bar{K} .

- The amount of machines is fixed at \bar{K} .
 - Capital is **exogenous!**
- The amount of workers is fixed at \bar{L} .
 - Labor is **exogenous!**
- This also means that the output is fixed $\bar{Y} = F(\bar{K}, \bar{L})$

Then, we can find the real factor prices:

- Use Eq. (1) to find the real wage.
- Use Eq. (2) to find the real rental price of capital.



Income Distribution

There are three different sources of income in our model:

- **Labor Income:** Workers receive $W \cdot L$
- **Capital Income:** Capital owners receive $R \cdot K$ by renting capital.
- **Firm's Profit:** $P \cdot Y - W \cdot L - R \cdot K$

How big is each of these three different sources?

- Without any further assumption, we can only pin down how much is the firm's profit.
- In order to do that, we will rely on the **Euler's Theorem**.

Theorem (Euler's Theorem)

If a production function has constant returns to scale, then:

$$Y = F(K, L) = MPL \cdot L + MPK \cdot K$$

Income Distribution

We can rewrite labor income using Eq. (1) as:

$$W \cdot L = P \cdot MPL \cdot L$$

We can rewrite capital income using Eq. (2) as:

$$R \cdot K = P \cdot MPK \cdot K$$

Let's plug these two in the expression for the firm's profit:

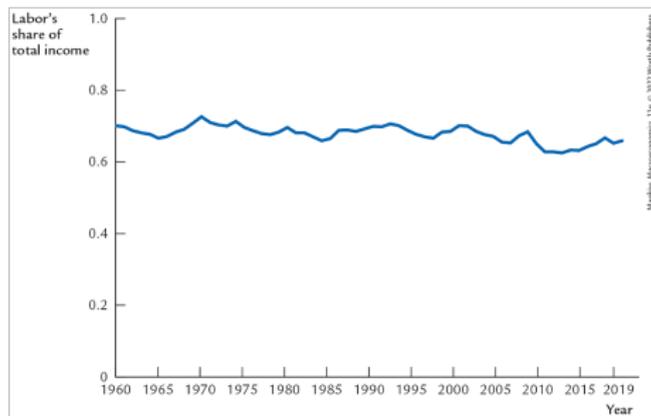
$$\begin{aligned} \text{Firm's Profit} &= P \cdot Y - W \cdot L - R \cdot K \\ &= P \cdot Y - P \cdot MPL \cdot L - P \cdot MPK \cdot K \\ &= P \underbrace{(Y - MPL \cdot L - MPK \cdot K)}_{=0 \text{ Euler's Theorem!}} \\ &= 0 \end{aligned}$$

Firm's profit is equal to 0!

Factor Shares - Data

Let's first examine the data to determine the characteristics of the factor shares, then return to our model.

The next figure shows the **ratio of labor income to total income** in the U.S. from 1960 to 2019.



Despite the many changes in the economy over the past six decades, **this ratio has remained about 2/3**.

- Even though the amount of labor and capital in the economy has changed a lot over time.

Cobb-Douglas production function

Paul Douglas, a senator from Illinois from 1949 to 1967, noticed this surprising fact in 1927.

Douglas asked **Charles Cobb**, a mathematician, what production function would lead to constant factor shares if factors always **earned their marginal products**.

Stated otherwise, Douglas wanted a production function that satisfies:

$$\text{Capital Income} = R \cdot K = P \cdot MPK \cdot K = \alpha \cdot PY$$

$$\text{Labor Income} = W \cdot L = P \cdot MPL \cdot L = (1 - \alpha) \cdot PY$$

for some **constant** $\alpha \in (0, 1)$, **no matter the values for K and L** . That means:

- A fraction α of total income is **capital income**. **ALWAYS!**
- A fraction $1 - \alpha$ of total income is **labor income**. **ALWAYS!**

Cobb-Douglas production function

Cobb, **the man, the myth, the legend**, showed that the function with these properties is:

$$F(K, L) = AK^\alpha L^{1-\alpha}$$

where $A > 0$ is a parameter that measures the **productivity** of the available technology.

This function became known as the **Cobb-Douglas production function**.

We will **assume** that the production function in **our model** is a **Cobb-Douglas**.

Cobb-Douglas: Properties

Constant Returns to Scale: $F(zK, zL) = zF(K, L)$

MPK:

$$MPK = \frac{\partial F}{\partial K} = A\alpha K^{\alpha-1}L^{1-\alpha} = \boxed{\alpha \frac{Y}{K}}$$

MPL:

$$MPL = \frac{\partial F}{\partial L} = A(1-\alpha)K^{\alpha}L^{-\alpha} = \boxed{(1-\alpha)\frac{Y}{L}}$$

Capital Income:

$$\text{Capital Income} = R \cdot K = P \cdot MPK \cdot K = \alpha PY$$

Labor Income:

$$\text{Labor Income} = W \cdot L = P \cdot MPL \cdot L = (1-\alpha)PY$$

Cobb-Douglas

What value of α should we pick?

- The graph we saw a few slides above showed that labor receives about $2/3$ of total income.
- Since profits are zero, that means **capital receives $1/3$ of total income**.
- Therefore, we should set $\alpha = 1/3$ in our model in order to replicate this result from the data.

We are done with describing how **production is determined** and how the **incomes from production are distributed!**

We will now describe how the output of our model is used!

How output is used

We will assume that our economy is a **closed economy**, with no friends = (

- A country that **does not trade** with other countries.
- That means **net exports is 0**.

Therefore, there are **three different uses** for the goods and services it produces:

- **Consumption C**
 - Households consume some of the economy's output.
- **Investment I**
 - Firms and households use some of the output for investment.
- **Government Expenditures G**
 - The government buys some of the output for public purposes.

The national income account identity becomes:

$$Y = C + I + G$$

Consumption

When we eat food or shop on Amazon, we are consuming some of the output of the economy.

In **US**, all forms of consumption together make up about **two-thirds of GDP**.

We will consider a simple story of consumer behavior in our model!

Here, **households**:

- Receives income from their labor and the rent of the capital they own.
 - **!** The income of the households is equal to the output of the economy Y .
- Pay taxes T to the government.
- Decides how much of their after-tax income ($Y - T$) to **consume** and how much to **save**.

We call $Y - T$: **Disposable income**.

We assume that **consumption** is a **function** of **disposable income**:

$$C = C(Y - T)$$

where $C(Y - T)$ is the **consumption function**.

Consumption

Marginal Propensity to Consume: MPC

- The amount by which consumption changes when disposable income increases by one dollar.
- The MPC is always between zero and one: $0 < MPC < 1$:
 - An extra dollar of income increases consumption but by less than one dollar.
 - Why? Because they save a portion of it.
- If $MPC = 0.7$, households spend 70 cents and save 30 cents for each additional dollar of income.
- Formally:

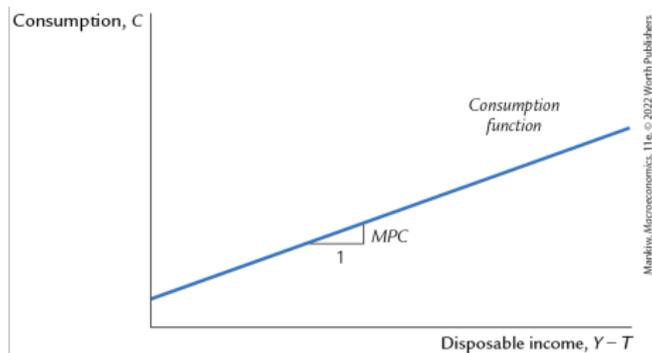
$$MPC = \frac{dC}{d(Y - T)}$$

Consumption

Example: Suppose the consumption function is $C = a + b \cdot (Y - T)$, for some constants a and b . Then:

$$MPC = \frac{dC}{d(Y - T)} = b$$

A beautiful plot to illustrate the consumption function in this example:



The **slope** of the consumption function is the **MPC**!

Investment

Both firms and households purchase investment goods:

- Firms buy structures (factories, office buildings) and equipment (machines, computers) and replace existing capital as it wears out.
- Households buy new houses.

Curiosity: in the US, total investment averages about 15% of GDP.

Investment depends on the interest rate:

- Interest rate measures the cost of the funds used to finance investment.
- Usually, we distinguish between two types of interest rates: nominal and real.
 - Nominal interest rate: The rate of interest that investors pay to borrow money. It is also the one usually reported.
 - Real interest rate: is the nominal interest rate corrected for the effects of inflation.
 - We will go back to this later in this course!
- The real interest rate measures the true cost of borrowing, thus, determines the quantity of investment.

Investment

We will assume that investment is a **decreasing function** of the **real interest rate** r !

$$I = I(r)$$

⚠ The real interest rate r is **not necessarily equal** to the rental price of capital R we saw before!

But why a decreasing function? Let me tell you a short story:

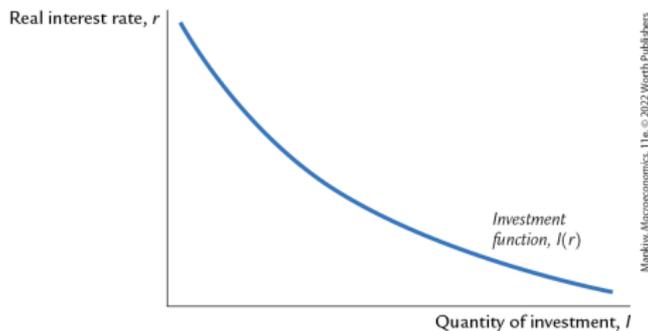
- Suppose a firm is considering whether it should build a \$1 million factory.
- The factory yield a return of \$100,000 per year (10%).
- The firm compares this return to the cost of borrowing the \$1 million.
 - If the real interest rate is **below 10%**, the firm **borrow**s the money and **make**s the investment.
 - If the real interest rate is **above 10%**, the firm **doe**s not build the factory.

Investment

- This is true even if the firm **uses its own funds** instead of borrowing:
 - **The firm can always deposit this money in a bank** and earn interest on it.
 - So if the interest rate is higher than the 10% return on the factory, they don't build it!

Note that as **the real interest rate rises**:

- **Fewer investment projects are profitable!**
- Therefore, the **demand for investment falls!**



Government purchases

The government spends on defense, education, roads, health and many other things.

In the US, government purchases of goods and services account for about 20% of GDP.

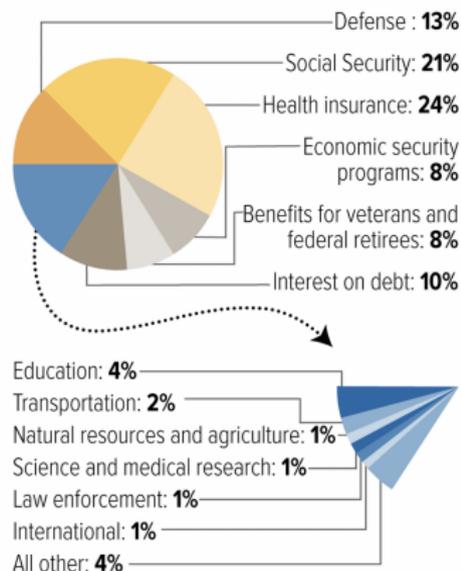
These purchases are **only one type** of government spending!

The other is **transfer** payments to households:

- Examples: **Public assistance** for the poor and **Social Security** payments for the elderly.
- Unlike government purchases, **transfer payments** are **not** made in **exchange for some of the economy's output** of goods and services.
-  Therefore, **they are not included in the variable G!**

Let's take a look at how the US government spends money, **including these transfers!**

Where Do Our Federal Tax Dollars Go?



Note: Percentages do not add to 100 percent due to rounding.

Source: 2023 figures from the Congressional Budget Office, May 12, 2023

CENTER ON BUDGET AND POLICY PRIORITIES | CBPP.ORG



This plot only shows the expenditures by the **federal government**.

- However, when we talk about government expenditures, we also include **states** and **cities** expenditures!

Fiscal Policy

Transfer payments **do affect the demand** for goods and services **indirectly**. They are the **opposite of taxes**:

- They **increase households' disposable income**, just as taxes reduce disposable income.
- An increase in transfer payments financed by an increase in taxes leaves disposable income unchanged.

Because of this, we will revise our definition of T :

$$T = \text{Taxes} - \text{Transfers}$$

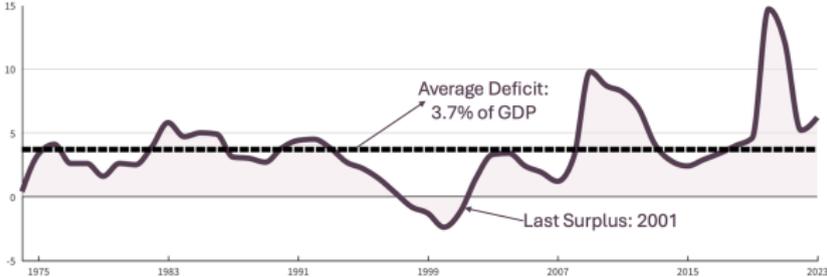
Fiscal Policy: The **choice** over the level of **government purchases and taxes**.

- If $G = T$, then government has a **balanced budget**.
- If $G > T$, then the government runs a **budget deficit**, which it funds by issuing government debt (**borrowing**).
- If $G < T$, then government runs a **budget surplus**, which it can use to **repay some of its outstanding debt**.

Fiscal Policy

Federal Deficits or Surpluses

Percentage of GDP

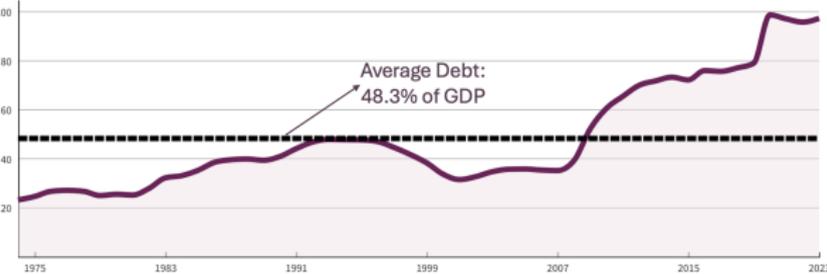


Source: Congressional Budget Office

Note: A positive value indicates a budget deficit and a negative value indicates a budget surplus.

Federal Debt Held by the Public, 1974 to 2023

Percentage of Gross Domestic Product



Source: Congressional Budget Office

Exogenous and Endogenous variables in our model

We **do not explain** the political process that sets **fiscal policy** in our model.

Therefore, both G and T will be **exogenous** and fixed at values \bar{G} and \bar{T} , respectively.

$$G = \bar{G}, \quad T = \bar{T}$$

Remember that K and L are also **exogenous**!

Then, the rest of the variables are **endogenous**: Y, C, I and r .

Our goal is to see how the **exogenous variables** affect the **endogenous variables**.

In order to do that, we need to **close the model**: Supply = Demand

Equilibrium in the Market for Goods and Services

The following equations summarize our description of the **demand** for goods and services Y^d :

$$Y^d = C + I + G$$

$$C = C(Y^d - T)$$

$$I = I(r)$$

$$G = \bar{G}$$

$$T = \bar{T}$$

The factors of production and the production function determine the **output supplied** Y^S in our model:

$$Y^S = F(\bar{K}, \bar{L}) = \bar{Y}$$

In equilibrium, **demand for goods** Y^d must be **equal** to the **supply of goods** \bar{Y}

Equilibrium in the Market for Goods and Services

Combining all the equilibrium conditions described in the last slide yields:

$$\bar{Y} = C(\bar{Y} - \bar{T}) + I(r) + \bar{G}$$

The **interest rate** r plays a key role: it has to **adjust** to ensure that the **demand for goods equals** the **supply**.

Let's go through a numerical example to see how to find the interest rate in equilibrium. Suppose:

- $Y = F(K, L) = K^{0.3}L^{0.7}$
- $K = 120, L = 120$
- $G = 10, T = 20$
- $C = 20 + 0.8 \cdot (Y - T)$
- $I = 20 - 100r$

Hint: Follow this order $Y \rightarrow C \rightarrow I \rightarrow r$

Equilibrium in Financial Markets: The Supply and Demand for Loanable Funds

How does the **interest rate** get to the level that **balances** the **supply** and **demand** for goods and services?

The best way to answer this question is to consider how **financial markets** fit into the story.

Rewrite the national income account identity as:

$$Y - C - G = I$$

The left-hand side is called **national saving**: S

$$S = Y - C - G$$

Note that: **Saving equals investment. Always!**

Private and Public Saving

We can split national saving into two parts:

- **Private saving (S^P)**: Private revenue (Y) minus private spending ($T + C$):

$$S^P = Y - T - C$$

- **Public saving (S^G)**: Government revenue (T) minus government spending (G):

$$S^G = T - G$$

- Note that:

- $S^G > 0$: budget surplus
- $S^G = 0$: balanced budget
- $S^G < 0$: budget deficit

Important: Private saving + Public saving = National saving!

$$S^P + S^G = Y - \cancel{T} - C + \cancel{T} - G = Y - C - G = S$$

Equilibrium in Financial Markets

The flows into the financial markets (private and public saving) must balance the flows out of the financial markets (investment).

Saving is the supply of loanable funds:

- Households lend their savings to investors.

Investment is the demand for loanable funds:

- Investors borrow from the public directly by selling bonds or indirectly by borrowing from banks.

The interest rate adjusts until the amount that firms want to invest equals the amount that households want to save.

Equilibrium in Financial Markets: Graphical Analysis

Let's recap the procedure to find r :

First, remember that: T , G , K and L are fixed and given! We denote the fixed values by \bar{K} , \bar{L} , \bar{T} and \bar{G} .

Second, we find Y , since we have \bar{K} and \bar{L} :

$$\bar{Y} = F(\bar{K}, \bar{L})$$

Third, since we have Y and T , we can find C :

$$C = C(Y - T) = C(\bar{Y} - \bar{T})$$

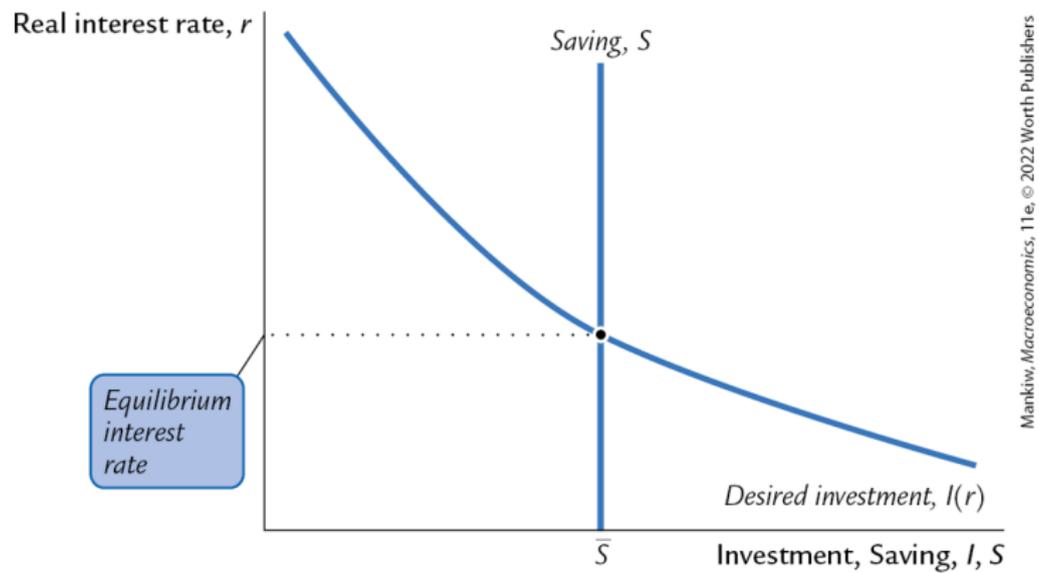
Finally, we find national saving S :

$$\bar{S} = \bar{Y} - C(\bar{Y} - \bar{T}) - \bar{G}$$

To find r , remember: **National Saving = Investment**.

$$\bar{S} = I(r)$$

Equilibrium in Financial Markets: Graphical Analysis



Working with Δ

We want to understand how **fiscal policy** can **affect** the **endogenous variables** of our model.

Before we do that, we need to understand how to find the **change** in some variable Y : ΔY .

Something is going to change in our model, and we want to find the **difference** between the variable Y **after** the change Y_a and **before** the change Y_b :

$$\Delta Y = Y_a - Y_b$$

Notation:

- A variable with subscript “b” means the value of that variable **before** the change.
- A variable with subscript “a” means the value of that variable **after** the change.

Working with Δ

I want to show you **two properties** that will make your life easier:

1. If $Y = X + Z$, for two other variables, X and Z , then:

$$\Delta Y = \Delta X + \Delta Z$$

2. If $Y = m \cdot X$, for some variable X and a **constant** m , then:

$$\Delta Y = m \cdot \Delta X$$

Proof first property:

- **Before** something changed: $X = X_b$ and $Z = Z_b \Rightarrow Y_b = X_b + Z_b$.
- **After** something changed: $X = X_a$ and $Z = Z_a \Rightarrow Y_a = X_a + Z_a$.

Then ΔY is:

$$\begin{aligned}\Delta Y &= Y_a - Y_b \\ &= X_a + Z_a - (X_b + Z_b) \\ &= X_a - X_b + Z_a - Z_b \\ &= \Delta X + \Delta Z\end{aligned}$$

Working with Δ

Proof second property:

- Before something changed: $X = X_b \Rightarrow Y_b = m \cdot X_b$.
- After something changed: $X = X_a \Rightarrow Y_a = m \cdot X_a$.

Then:

$$\Delta Y = Y_a - Y_b = mX_a - mX_b = m(X_a - X_b) = \boxed{m \cdot \Delta X}$$

Note the **similarity** between these and the properties of **derivatives**!

Let me remind you of **two** other **trivial** properties:

- If **two variables are equal** to each other: $Y = X$, then:

$$\boxed{\Delta Y = \Delta X}$$

- If Y is **not affected by the change**, then:

$$\boxed{\Delta Y = 0}$$

An increase in G

Suppose that the government **increases G** .

- **Output doesn't change** since K and L are fixed.
- **Taxes are also fixed** at \bar{T} , so it doesn't change either.
- This means that **consumption doesn't change**, since $C = C(\bar{Y} - \bar{T})$
- Remember that: $Y = C + I + G$. Using the properties we just saw:

$$\begin{aligned} Y &= C + I + G \Rightarrow \\ \underbrace{\Delta Y}_{=0} &= \underbrace{\Delta C}_{=0} + \Delta I + \Delta G \Rightarrow \\ \boxed{\Delta I} &= -\Delta G \end{aligned}$$

The increase in G must be met by an equal **decrease** in I .

In order to decrease I , the interest rate r must **increase**.

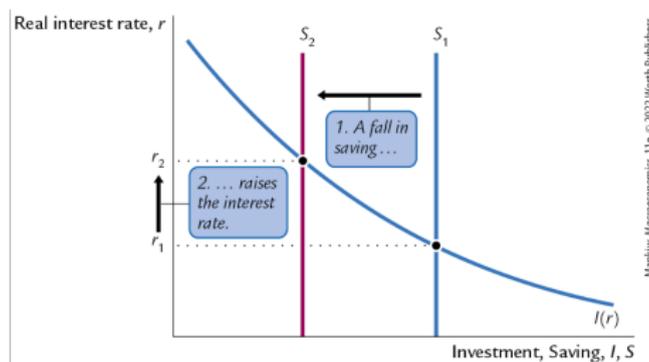
An increase in G

Conclusion: An increase in G

- Increase the interest rate r .
- Decrease investment I .

That's why government purchases are said to crowd out investment.

Graphically:



Change in Taxes

We want to understand what happens if we change taxes by ΔT .

- The remaining exogenous variables are fixed:

- $Y = \bar{Y}$.

- $G = \bar{G}$.

We will assume the consumption function is:

$$C = a + b \cdot (Y - T)$$

First, we write an expression for national saving:

$$S = \bar{Y} - C - \bar{G}$$

$$S = \bar{Y} - [a + b(\bar{Y} - T)] - \bar{G}$$

$$S = \bar{Y} - a - b\bar{Y} + bT - \bar{G}$$

Then, using the properties we just saw:

$$\Delta S = b \cdot \Delta T$$

Change in Taxes

Since $I = S$, then $\Delta I = \Delta S$:

$$\Delta I = \Delta S = b \cdot \Delta T$$

Therefore, a **decrease** in taxes yields:

- A **decrease in investment**, since $\Delta T < 0$ and $b > 0$.
- An **increase** in the interest rate r .

Example: Suppose that consumption is a linear function of disposable income and that the marginal propensity to consume is 0.8. The investment function is given by $I(r) = 10 - 100 \cdot r$. If the government decides to decrease taxes by 10, find:

- The impact on national saving ΔS .
- The impact on the interest rate Δr .

Changes in Investment Demand

Two examples of when the investment demand function can change:

- Technological innovation:

- The invention of a new technology, such as the railroad or the computer.
- Before, a firm or household can take advantage of the innovation, it must buy investment goods.

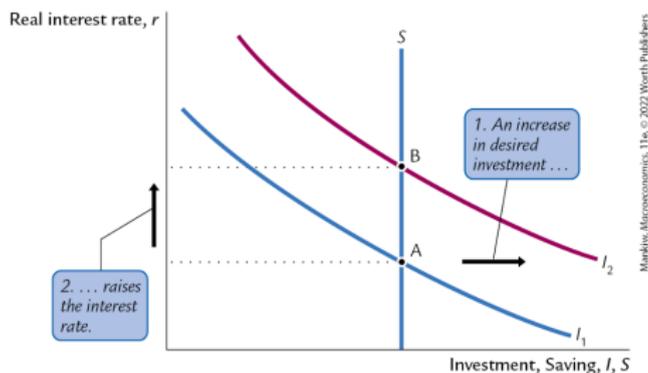
- Change in tax laws:

- The government increases personal income taxes and uses the extra revenue to provide tax cuts for those who invest in new capital.

In both scenarios, at any given interest rate, the demand for investment goods (and also for loanable funds) is higher.

Changes in Investment Demand

What are the consequences of this change in the investment demand function?



The **surprising result**:

- Remember that the **level of savings is fixed!**
- Then, the **equilibrium amount of investment can't change!**
- A shift in the investment demand function **merely raises the equilibrium interest rate.**

This doesn't sound right! Can we fix this?

Change in Investment Demand

The problem is our **consumption function** (and its flip side, saving) **doesn't depend on the interest rate!**

The **interest rate** is also the **return on savings!**

- A higher interest rate might increase your incentives to reduce consumption, thus increasing savings!
- If we allow **consumption** to depend (negatively) on the **interest rate**, the **saving schedule would be upward-sloping** rather than vertical.

If the **saving schedule slopes upward**, an increase in investment demand raises both the **interest rate** and the equilibrium **quantity of investment**.

