Inflation

Rodrigo Pacheco

Columbia University

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Today's plan:

- What is Inflation?

- What causes Inflation?
  - The Quantity Theory of Money.

- What are the effects of inflation?
  - Inflation tax.
  - Real and nominal interest rates and the Fisher equation.
  - Ex ante versus ex post interest rates.

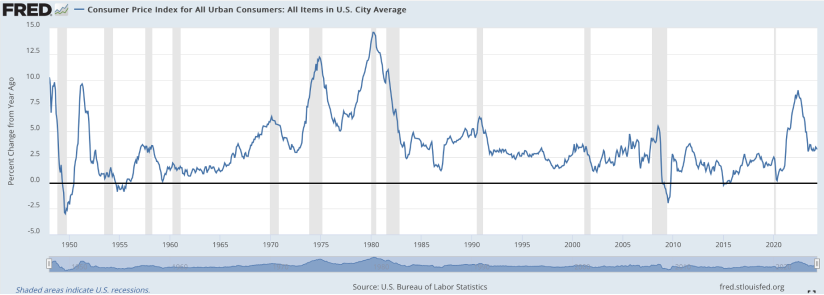
- What are the social costs of inflation?

Big Picture

Inflation is the overall increase in prices of goods and services in an economy over time.

- Overall is key here!
- An increase in only one or two prices is not inflation!
- Inflation is more about the value of money than about the value of goods.
- When there is inflation, money loses value/purchasing power.

Let's take a look at the history of inflation in the US:



Understanding the recent monetary history

To deal with inflation, central banks were made independent:

- They are not under the direct influence of government.
- The President appoints the Fed Chair (now Powell) for a sufficiently long term.
- This way, the bank is out of reach for everyday politics.
- This is the case in the US, and many other countries.

What explains the decision to adopt central bank independence?

To answer this question we need to understand:

- How inflation is determined in the long run.
- Why it is a problem.

Big Picture

In this lecture we will examine the classical theory of inflation:

- Prices are assumed to be flexible!
- This is a good theory for the long run!
- In the short run, prices are sticky.
- We will study the short run later in this course.

What causes inflation in the long run?

The Quantity Theory of Money will explain this!

The Quantity Theory of Money

Building block on the Quantity Theory of Money: velocity of money.

Velocity of money:

- The number of times the average dollar bill is used to buy goods and services in a given period.
- Informally, the number of times the average dollar bill changes hands in a given time period.

Example: Suppose in 2021 we had $500 billion in transactions and the money supply was $100 billion.

- A dollar is used in 5 transactions
- Then, velocity is 5.

If we could measure transactions, we could calculate velocity easily.

Unfortunately, we cannot measure transactions directly.

The Quantity Theory of Money

We will use nominal GDP as a proxy for transactions!

- They are related: the more an economy produces the more goods and services are bought.
- Not perfect: when you sell your used car, money was used, but GDP did not increase.

The Quantity Theory of Money is based on the Quantity Equation:

$$\text{Money} \times \text{Velocity} = \text{Price} \times \text{Output}$$

$$\boxed{M \times v = P \times Y}$$

- M is the money supply.
- v is the velocity of money.
- P is the GDP Deflator.
- Y is the Real GDP.
  - Equivalently, $P \cdot Y$ is the nominal GDP.

Real Money Balances

Real money balances M/P: the quantity of money in terms of the quantity of goods and services it can buy.

It measures the purchasing power of the stock of money.

Example: Suppose an economy that produces only cookies.

- The money supply is \$100.
- The price of a cookie is \$2.
- Then, the real money balances are $100/2 = 50$. We can buy 50 cookies with the money available in the economy.

Money demand function: $(M/P)^d$ it describes the quantity of real money balances people want to hold.

Example:

$$(M/P)^d = \kappa Y$$

where κ is a constant that tells us how much money people want to hold for every dollar of income.

The money Demand Function and the Quantity Equation

We will interpret the Quantity Equation in terms of this money demand function.

The equilibrium condition says that demand for real money balances equals the supply!

$$(M/P)^d = (M/P)$$

Using the money demand function:

$$M \cdot 1/\kappa = P \cdot Y$$

Comparing this with the Quantity Equation:

$$M \cdot v = P \cdot Y$$

We can see that $v = 1/\kappa$

Let's try to get some intuition from this.

The Quantity Theory of Money

Suppose you want to hold a lot of money:

- This means κ is high.
- A lot of cash on hand means you are not using it to buy things $\Rightarrow$
- Money is not changing hands frequently $\Rightarrow$
- Velocity is low.

But, when would you want to hold a lot of cash?

- When money has a lot of purchasing power!
- I.e. you can buy a lot of things with a dollar $\Rightarrow$
- Prices are low! P is low.

Conclusion: when prices are low, velocity is low!

You can see this using the quantity equation:

$$M \cdot v = P \cdot Y$$

If we fix M and Y, then a low P means a low V!

QTM and Inflation

We want an expression for the inflation rate as a function of the growth rate of the money supply, the growth rate of real GDP and the growth rate of velocity.

Write the Quantity Equation for two consecutive time periods: $t-1$ and t:

$$M_t v_t = P_t Y_t$$

$$M_{t-1} v_{t-1} = P_{t-1} Y_{t-1}$$

Dividing the first equation by the second:

$$\boxed{\frac{M_t}{M_{t-1}} \frac{v_t}{v_{t-1}} = \frac{P_t}{P_{t-1}} \frac{Y_t}{Y_{t-1}}} \tag{1}$$

The inflation rate π_t is the percentage change in the price level:

$$\boxed{\pi_t = \frac{P_t - P_{t-1}}{P_{t-1}}}$$

Rearranging we get: $\frac{P_t}{P_{t-1}} = 1 + \pi_t$

QTM and Inflation

We define the growth rate of output between periods $t-1$ and t as g_t^Y:

$$g_t^Y = \frac{Y_t - Y_{t-1}}{Y_{t-1}} \implies \frac{Y_t}{Y_{t-1}} = 1 + g_t^Y$$

Similarly, g_t^M is the growth rate of money stock between periods $t-1$ and t:

$$g_t^M = \frac{M_t - M_{t-1}}{M_{t-1}} \implies \frac{M_t}{M_{t-1}} = 1 + g_t^M$$

and g_t^v is the growth rate of velocity of money between periods $t-1$ and t:

$$g_t^v = \frac{v_t - v_{t-1}}{v_{t-1}} \implies \frac{v_t}{v_{t-1}} = 1 + g_t^v$$

Money Prices and Inflation

Substituting these in Equation (1):

$$\left(1+g_t^M\right)\left(1+g_t^v\right)=\left(1+\pi_t\right)\left(1+g_t^Y\right)$$

Taking logs on both sides:

$$\ln\left(1+g_t^M\right)+\ln\left(1+g_t^v\right)=\ln\left(1+\pi_t\right)+\ln\left(1+g_t^Y\right)$$

Use the beautiful approximation $\ln(1+x)\approx x$ for small x:

$$g_t^M+g_t^v=\pi_t+g_t^Y$$

Finally, isolate the beautiful inflation rate:

$$\boxed{\pi_t=g_t^M+g_t^v-g_t^Y}$$

The assumption of constant velocity

The Quantity Theory of Money assumes that velocity is constant.

Then, the growth rate of velocity is zero!

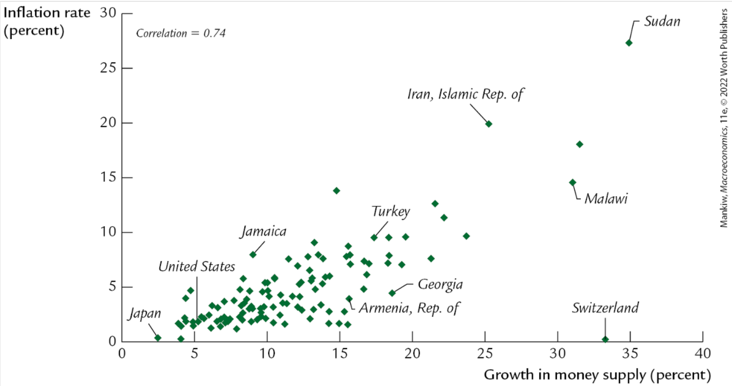
Using this in our last equation yields the beautiful prediction of the QTM:

$$\boxed{\pi_t = g_t^M - g_t^Y}$$

This is a very powerful equation:

- If the money supply grows faster than GDP, inflation will be positive.
- The central bank has ultimate control over inflation, since it controls the money supply!

Confronting the quantity theory with data

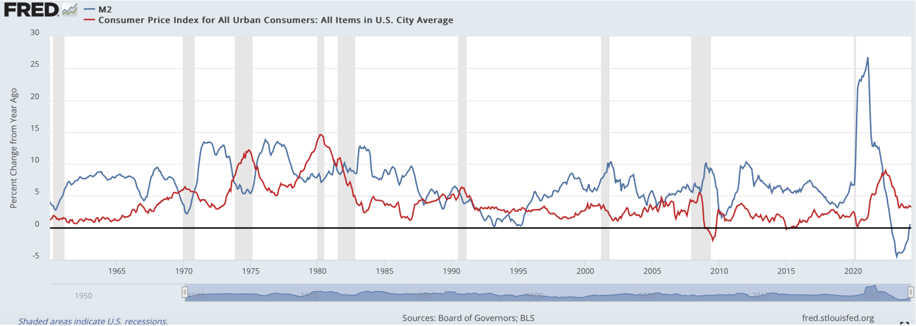


- Each point represents a country.
- The x-axis is the average growth rate of money supply, from 2007-2019.
- The y-axis is the average inflation rate, from 2007-2019.

Conclusion: the QTM seems to be a good approximation of reality in the long run.

Confronting the quantity theory with data

What about the short run?



- It seems that in the short run inflation and money growth appear negatively correlated!
- The causality might be reversed in the short run:
  - When inflation rises, or is expected to rise, the Fed cuts back on money growth, and vice versa.

Seigniorage

Government can finance its spending in three ways:

- Taxes
- Borrowing
- Printing money

Seigniorage: the revenue the government raises by printing money.

When the government prints money to finance expenditure, it increases the money supply.

The increase in the money supply, in turn, causes inflation, as we saw.

This is like a tax on the holders of money.

- Although you are not paying the tax directly, the action of the government is reducing the amount of goods you are able to buy!
- This is called the inflation tax.

Let's see how this works with an example

Seigniorage Example

Suppose we live in a world where the only good produced is cookies and Rodrigo is the only consumer:

- The economy produces 100 cookies per year.
- The money supply is \$100.
- The price of a cookie is \$1.
- If the government doesn't hold any money, Rodrigo can buy all 100 cookies with his \$100.

The QTM tells us that the velocity of money is 1:

$$v = \frac{P \cdot Y}{M} = \frac{1 \cdot 100}{100} = 1$$

Now suppose the government decides to print \$20 more in order to finance its spending and the number of cookies produced doesn't change.

How many cookies can Rodrigo buy now?

Seignorage Example

The QTM tells us that the velocity of money is still 1, since it is assumed to be constant.

The money supply is now \$120.

How much is the price of a cookie now?

$$M \cdot v = P \cdot Y \implies P = \frac{M \cdot v}{Y} = \frac{120 \cdot 1}{100} = 1.2$$

Rodrigo can now buy only $100/1.2 \approx 83$ cookies.

Although he has the same amount of money and didn't pay any taxes, it is as if he paid a tax of 100 - 83 = 17 cookies!

Inflation and Interest rates

We will define two interest rates:

- Nominal interest rate.
- Real interest rate.

Nominal interest rate i: the rate of interest that is stated in a contract/loan.

Real interest rate r: the difference between the nominal interest rate and the inflation rate π.

The relationship between these three variables can be written as:

$$\boxed{r = i - \pi}$$

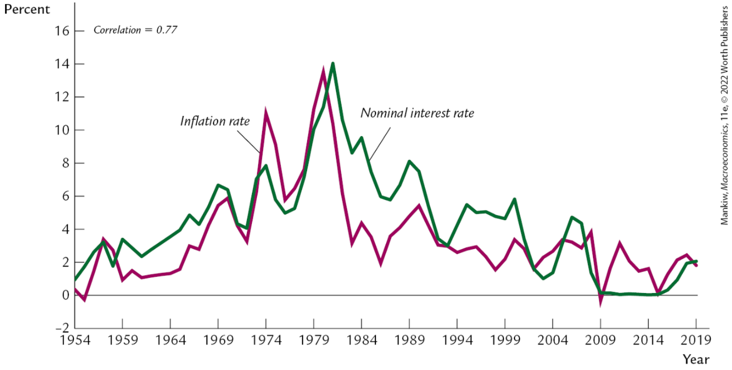
Rearranging this equation we get the famous Fisher Equation:

$$\boxed{i = r + \pi}$$

The one-for-one relation between the inflation rate and the nominal interest rate is called the Fisher effect.

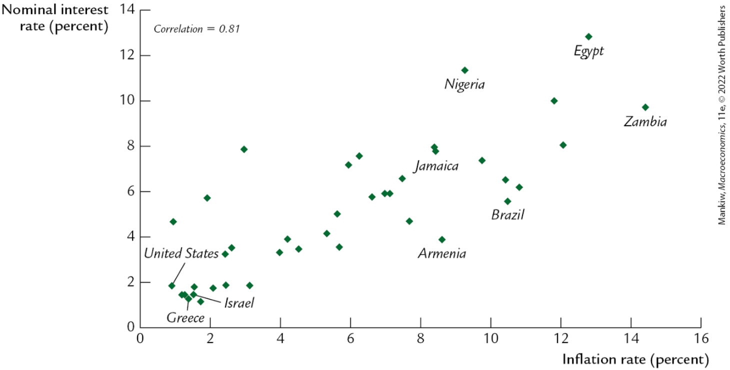
The Fisher Equation: confronting the theory with the data

U.S. inflation and nominal interest rates, 1955-2020



The Fisher Equation: confronting the theory with the data

Inflation and nominal interest rates in 48 countries



This plot shows the average nominal interest rate and the average inflation rate in 100 countries during the period 2000 to 2013.

The Fisher Effect

We will bring the QTM and the Fisher equation together with an example.

Example: Suppose the Fed is increasing the money supply by 10% per year, Y is growing 2% per year, and $r = 4\%$.

1. What is the inflation rate?
2. What is the nominal interest rate?
3. Now suppose the Fed increases the money growth rate by 2 percentage points per year.
   3.1 What happens to the inflation rate?
   3.2 What happens to the nominal interest rate?

Ex Ante and Ex Post Interest Rates

When a borrower and lender agree on a nominal interest rate, they do not know what the inflation rate over the term of the loan will be.

Therefore, we will define two types of real interest rates:

- Ex ante real interest rate.
- Ex post real interest rate.

Ex ante real interest rate:

- The real interest rate that the borrower and lender expect when the loan is made.
- We will refer to expected inflation as $E\pi$.
  - This is how much you expect inflation to be in the future.
- The ex ante real interest rate is $r = i - E\pi$.

Ex post real interest rate:

- The real interest rate that is actually realized over the term of the loan.
- The ex post real interest rate is $r = i - \pi$.
  - Where π is the actual inflation rate over some period.

Revisiting the Fisher effect

People make decisions about saving and investment before inflation is known.

Therefore, the ex ante real interest rate is the relevant one!

What about the Fisher Effect? We need a small modification:

- Actual inflation is not known when the nominal interest rate is set!
- Therefore, the nominal interest rate cannot adjust to actual inflation.

The Fisher effect is better written as:

$$\boxed{i = r + E\pi}$$

Demand for Money and nominal interest rates

The QTM assumes that the demand for money is proportional to the level of income.

That's a good starting point.

In reality, the demand for money is also influenced by the nominal interest rate:

- The money you hold in your wallet does not earn interest.
- If, instead, you used the money to buy government bonds, you would earn the nominal interest rate.
- That's why the nominal interest rate is the opportunity cost of holding money.

Based on this, we will update our money demand function to include the nominal interest rate.

$$\boxed{(M/P)^d = L(i, Y)}$$

Demand for Money and nominal interest rates

$L(i, Y)$ is a function of the nominal interest rate and income:

- L is inspired by the fact that the money is the most liquid asset.
- The higher the interest rate i, the lower L.
    - You are losing a lot of interest by holding money, so you want to hold less of it!
- The higher the income Y, the higher L.
    - You have more money, so you want to hold more cash.

Let's combine this with the Fisher equation and see the power of expectations.

Replace i by $r + E\pi$ in the money demand function:

$$(M/P)^d = L(r + E\pi, Y)$$

The implications of this equation are a bit different from the QTM.

The role of Expectations

The QTM stated that the today's money supply determines today's price level.

The Fisher equation and the money demand function tell us that expected inflation also matters!

Let's see the implications of this using an example.

Suppose people expect inflation to be higher in the future. What happens today?

$$M/\uparrow P =\downarrow L(r + \uparrow E\pi, Y)$$

Effects on the right side of the equation:

- Output today is already produced. So it doesn't change.
- The real interest rate is also fixed, since it is determined by the supply and demand for loanable funds.
- The nominal interest rate is higher, because $E\pi$ is higher $\Rightarrow$
- Demand for real money balances is lower.

The role of Expectations

$$M/\uparrow P =\downarrow L(r+\uparrow E\pi, Y)$$

The right side decreases! Then, the left side must also decrease:

- The money supply today doesn't change. Fed already chose it.
- The price level today must increase, then.
- This means inflation today is higher!

Conclusion: expecting inflation in the future causes inflation today!

- The price level depends not only on today's money supply (QTM) but also on the money supply expected in the future (via expected inflation)

This explains:

- Why central banks are so concerned about anchoring inflation expectations.
- Why the credibility of central banks is so important.

Fed and the Expectations channel

Nowadays, Fed policy tries more actively to affect the market's expectations.

Forward Guidance: Fed actually announce a "path" for what they will do in the future.

New York Times, Jun 14th:

The Fed Holds Rates Steady and Predicts Just One Reduction This Year

Federal Reserve officials signaled that interest rates could stay higher this year as policymakers pause to ensure they've stamped out inflation.

This is all because central banks have understood the crucial role of expectations. Managing people's expectations can be of help!

We will revisit this when we talk about the short run.

The Social Costs of Inflation

A common misperception is that inflation reduces real wages.

This is true only in the short run, when nominal wages are fixed by contracts.

- In the short run people are upset!

However, in the long run nominal wages do respond to inflation and there is, on average, real wage growth.

Remember our discussion in the closed economy model:

- The real wage is determined by labor supply and the marginal product of labor (labor demand).

Here, we want to understand the costs of inflation in the long run.

The Social Costs of Inflation

We will divide the costs of inflation into two categories:

Costs of expected inflation: E.g., you know that your money will lose purchasing power at a rate of 5% per year.

- Shoe-leather costs.
- Menu costs.
- Relative prices distortions.
- Tax distortions.

Costs of unexpected inflation: You don't know how much inflation will be in the future.

- Wealth redistribution.

Costs of Expected Inflation

Imagine we live in a world where inflation is 6% every year, no matter what.

What would be the social costs of this?

Shoe-leather costs:

- As we saw, inflation leads to a higher nominal interest rate.
- This increases the opportunity cost of holding money: the interest you could have earned.
- People will go to the bank more often in an attempt to maximize their interest earnings.
- Going to the bank more often is costly!
- This is called shoe-leather costs: walking to the bank more often causes ones' shoes to wear out more quickly.
- Arguably a small cost.

Costs of Expected Inflation

Menu Costs:

- High inflation induces firms to change their posted prices more often.
- This is also costly: think about the cost of printing new menus, or changing prices on a website.
- We call this Menu Costs.

Relative price distortions:

- Firms facing menus costs change prices infrequently.
- E.g. a firm could only change prices once a year, in January.
- In the first half of the year, sales will be too low.
  - Prices are too high relative to other goods, since the firm needs to compensate for the expected inflation.
- In the second half of the year, sales will be too high.
  - Prices are too low relative to other goods, since the firm can't adjust prices.
- This yields variability in relative prices $\Rightarrow$ inefficiency in the allocation of resources.

Costs of Expected Inflation

Tax Distortions:

- Suppose you buy some stock today at \$100 per share and sell it a year from now at \$110 per share.
- Inflation in the meantime was 10%.
- There was no real gain, yet you will pay taxes on the \$10 “gain”.
- If the tax rate is 30%, you will pay \$3 in taxes.
- The result: a real loss of 3%.

Costs of Unexpected Inflation

Unexpected inflation redistributes wealth among the population.

Ex post someone likes it and someone dislikes it: a zero-sum game. Let's find out who wins and who loses.

Most loan agreements specify a nominal interest rate.

If actual inflation turns out to be higher than expected, who wins?

Let'see an example! Suppose an economy that only produces insomnia cookies:

- The debtor borrows $100 from the creditor.
- They agree on a nominal interest rate of 50% per year.
- The price of the insomnia cookie today is 1 dollar.
- They expect the price of the insomnia cookie to be 1.1 dollar in the future.
- It turns out the actual price of the insomnia cookie is 1.2 dollar in the end.

Costs of Unexpected Inflation

The debtor:

- Receives $100/1$ insomnia cookies today.
- Will have to pay $100 \cdot 1.5 = 150$ dollars in the future.
- He expects to pay $150/1.1 = 136.36$ insomnia cookies in the future.
- Ends up paying $150/1.2 = 125$ insomnia cookies.
- He is paying fewer cookies than expected.
- He is so happy.

The creditor:

- Lends $100/1$ insomnia cookies today.
- He will receive $100 \cdot 1.5 = 150$ dollars in the future.
- Expects to receive $150/1.1 = 136.36$ insomnia cookies in the future.
- Ends up receiving $150/1.2 = 125$ insomnia cookies.
- He is receiving fewer cookies than expected.
- He is so sad.

Costs of Unexpected Inflation

Conclusion:

$$\text{Actual inflation} > \text{Expected inflation} \Rightarrow \text{Debtor wins, Creditor loses}$$

Always think in terms of real interest rate: If actual inflation is higher than expected inflation:

- The ex post real interest rate is lower than the ex ante real interest rate.
- The debtor is happy because he pays back a lower real interest rate.
- The creditor is sad because he receives a lower real interest rate.

In a sense, unexpected inflation is a redistribution of wealth between creditors and debtors.

Arguably, the costs of unexpected inflation are the most important.

Curious note: countries with high average inflation also tend to have inflation rates that change greatly from year to year.

A Benefit of Inflation

Some economists believe that a little bit of inflation, 2 or 3 percent per year, can be a good thing. Why?

Because of nominal wage rigidity.

- Firms are reluctant to cut their workers' nominal wages.
- Workers also don't like nominal wage cuts.

Remember the perfect world we studied in the closed economy model:

- Real wages adjust so that the labor market is in equilibrium:
    - Labor supply equals labor demand.
- The condition is given by: $W/P = MPL$.

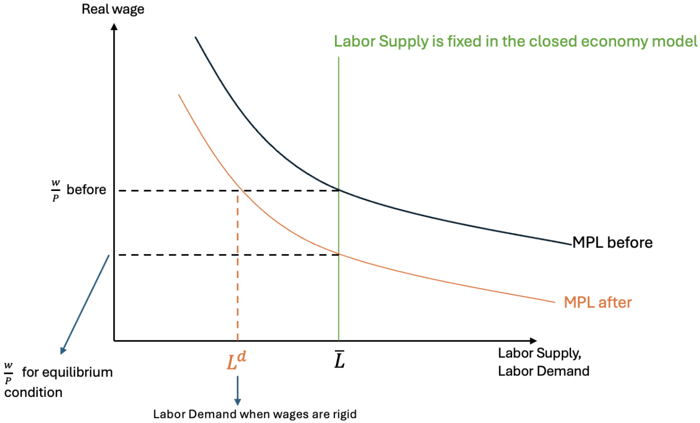
Now, if MPL falls, say because productivity falls (the parameter A in the Cobb-Douglas decrease), W/P must fall, to keep the labor market in equilibrium.

If there's no inflation, P is constant, then W must fall.

But if firms are reluctant to cut W, labor market will not be in equilibrium!

A benefit of Inflation

Graphically:



$\bar{L} - L^d$: Unemployment

A benefit of inflation

Suppose in our example that we need to cut real wages by 2% to keep the labor market in equilibrium.

If we have a bit of inflation, W/P can fall without W falling!

- A 2% wage cut in a zero-inflation world is, in real terms, the same as a 3% raise with 5% inflation.
- But the second makes workers happier!

This finding suggests that some inflation may make labor markets work better!

Inflation "greases the wheels" of labor markets.

Classical Dichotomy

In the closed economy model, we explained many macroeconomic variables.

All of them fit into the category of real variables: measured in physical units.

- Some of them were quantities: GDP and capital stock, for example.
- Some of them were relative prices: the real wage, for example.
    - Recall the real wage is the wage in terms of output.

When we talked about inflation and the monetary system, we mainly focused on nominal variables:

- Measured in money units.
- E.g.: the price level, the nominal wage, the nominal interest rate.

Classical Dichotomy

Surprisingly, we explained real variables without introducing nominal variables in the closed economy model.

We call this separation of real and nominal variables the classical dichotomy.

- This allows us to examine real variables without worrying about nominal ones.
- In classical economic theory, changes in the money supply do not influence real variables!

Monetary neutrality: the irrelevance of money in the determination of real variables.

- This is a good approximation in the long run, but not in the short run.